A

TREATISE

CONTAINING

The DESCRIPTION and USE

Of a ~~NEW and~~ CURIOUS

QUADRANT,

MADE and FINISHED

By the Masterly Hand of that Excellent MECHANIC,

JOHN ROWLEY;

For Taking of ALTITUDES,

And for Solving various MATHEMATICAL PROBLEMS in

Geometry, Navigation, Astronomy, *&c.*

Some of them by a bare INSPECTION of the INSTRUMENT, and others by easy OPERATIONS on it.

Studiously adapted to the meanest Capacities.

To which are prefixed,

An ALPHABETICAL EXPOSITION of the Necessary TERMS of ART, and a PLATE of the INSTRUMENT.

By *T. W.* F. R. S.

LONDON:

Printed for R. and J. DODSLEY, in *Pall-Mall.*

MDCCLVI.

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[illegible]

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QUADRANT,

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[illegible]

And for Solving various Mathematical Problems in

[illegible]

[illegible]

[illegible]

[illegible]

An Arrangement [illegible] of the [illegible] Terms of Art, and a TABLE of the Instrument.

[illegible]

LONDON:

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[illegible]

PREFACE.

THE Quadrant *(the subject of the present Treatise) was constructed by the celebrated Mr.* John Rowley, *at a time when he was just out of his apprenticeship; and, upon this occasion: His master (an excellent workman) complained that the Quadrant of Mr.* Collins *was crowded with such a number and variety of lines and arcs, both backside and foreside, some necessary, some unnecessary, and withal, that they were put so close together, that the eye was misguided and perplex'd in tracing them; and, after all, that the Quadrant was, in several respects, defective; and, therefore, he wished that some steady hand could be found to make a new one according to his directions, which Mr.* Rowley *undertook, and performed in such a masterly manner in all its parts, that not one stroke or division is amiss, displaced or disproportioned in the whole. And, as he was pleased with his own work, and met with it accidentally after his master's death, he bought it, recommended it, and sold it to me; a ~~loose~~ print or impression of which accompanies this treatise.*

To order and prepare this print for common use, it must be severed from ye *~~be~~ pasted on a smooth board, framed, fitted, and sized to it,* book & *with an handle to take off or put on at pleasure.*

But,

But, if it is to be used for taking heighths, then two sights made of brass or silver, are to be put on two grooves, to be affix'd to the line of tangents, according to the marks thereon for that purpose, and so to be mounted on a pedestal, as other Quadrants usually are, and thence taken off again for common use, when the altitude is observed; of which, however, more at large, when I come to describe the Apparatus of the Quadrant.

The Solutions of many Problems in the Mathematics are found, on this Quadrant, by a bare Inspection, with the help or apposition only of the thread upon it; and all the rules that are given in the books, particularly in Mr. Hodgson's System of Mathematics, *for the resolution of right lined, oblique, or spherical triangles, are exactly and critically answered, in practice, upon this Quadrant, as far as the divisions or graduations on it can admit; that is, as far as degrees and minutes of a degree. But, as the compass of the Quadrant will not allow of any sub-divisions to seconds of minutes, these are therefore not to be expected in this limited instrument; nor are they to be found in Mr.* Collins's *or Mr.* Sutton's *Quadrant, or any other of a foot radius: Nor, after all, is it necessary that it should comprehend seconds, unless in high calculations; and in such cases recourse must be had to the common tables of logarithms or artificial numbers, particularly to those of logistic logarithms extending to seconds.*

But, as the lines of sines, tangents and secants on this Quadrant, are framed by, and adapted to the tables of logarithms,

rithms, ſo far as to minutes of a degree, and the problems required to be ſolved, in common practice, are thus far eaſily reſolved by the Quadrant, it may therefore be made uſe of with great eaſe and certainty, inſtead of arithmetical or logarithmetical calculation; or, at leaſt, the work performed, on the Quadrant, may be of excellent uſe, to confirm the truth of the calculation, or ſhew its faults: and no man is ſo ſure of his ſingle calculation, as he may be by a joint and concurring teſtimony, both of the one and the other method.

Gunter's Scale *is a valuable inſtrument, and formed for anſwering the like uſes and purpoſes, as this of the preſent Quadrant; but yet it does not come up to the perfection of this Quadrant, the ſame being in ſome caſes defective; particularly where Altimetria, or the taking of heights, or angles in ſurveying, are required: both which are ſupplied by Mr.* Rowley's *Quadrant, which renders it preferable to thoſe inſtruments, as it ſupplies their defects, and anſwers all their purpoſes.*

Since Trigonometry is a neceſſary part of Geometry, it is therefore proper, in order to ſhew its connection with the Quadrant, to apply the rules given in the books, to examples, in practice on the Quadrant; ſo that, by comparing the rules with the practice, the reader may judge for himſelf, whether the rules and practice agree or diſagree.

Mr. Hodgſon, *in the firſt volume of his* Syſtem of Mathematics, *has demonſtrated the truth of ſeveral Theorems and Problems, neceſſary to the ſolution of ſeveral caſes of*

right and oblique angled plane triangles; and, in his second volume, has laid down the rules that are proper for the solution of spherical triangles, and when this foundation was laid by Mr. Hodgson, *he proceeds, particularly, to solve by these rules, the several cases of right lined and spherical triangles, and then applies the same to Navigation, Astronomy,* &c.

Now, since these rules have been given and proved by Mr. Hodgson, *it is needless, and would be impertinent, upon the present occasion, to repeat the same proofs; because the subject in hand is only to shew, that by the Quadrant those rules can be, and are, exactly answered and complied with.*

To accomplish this end, I shall, so far as is necessary to the present design, shew, in practice, the exact correspondency of the Quadrant to the rules given by Mr. Hodgson *for solving many cases in Trigonometry, Navigation, Astronomy,* &c. *and shall, in each case, set down the rules themselves, and refer, in the margin or text, to the pages in Mr.* Hodgson's *System, where they are demonstrated.*

I must here take notice, that the line of tangents on the Quadrant, as well as that on Gunter's *Scale, is fitted to the rest of the lines on this, as well as that instrument, and so goes no further, in this, than forty-five degrees.*

But

P R E F A C E.

But it happens ſometimes, that higher tangents are required, and more particularly in aſtronomical caſes; and this has made it neceſſary, wherever that is the caſe, to ſhew how the difficulty occurring from it may be removed, which has been the occaſion of exhibiting, in this treatiſe, more caſes than otherwiſe needed to have been, purpoſely, that no difficulty might appear, but at the ſame time be cleared; ſuch as theſe you will find in pages 54, 55, 62, 67, 74, 83, 85, 91, 96, &c.

In other caſes, particularly in the operations by ſines and by equal parts, I have ſhewn the practice at large, ſo often, till I found there could be no further need of it, and then I break off by ſaying, this rule or this problem is work'd in the common way.

It may be objected to this treatiſe, that it is drawn out into an unreaſonable length, by repeated caſes and operations, when references, to the like caſes, might have ſhortened it. This is true; but then I ſay, it would not have ſaved the reader the trouble of going back, upon every ſuch occaſion, to the place referred to; ſo that in truth, he would ſpend more time, that way, than this; and beſides, his thoughts, in that way, would have been ſuſpended and interrupted, by being under a neceſſity of leaving off, to look after the parallel caſe.

As ~~definitions of~~ ſome technical terms are frequently made uſe of in this diſcourſe, I have, for the eaſe of ſuch readers

as are not conversant in them, given such definitions of them *as are the most pertinent and proper for the present purpose; and have digested them, alphabetically, that so he may have resort thereto readily, without loss of time. And I chose to do this, at the beginning of the work, rather than to be breaking off the course thereof, by the interposition of such definitions. But, as to such readers as are learned in the Mathematics, these may, if they please, pass over the Definitions and Introduction too, and so go to the Description of the Quadrant, its Apparatus, Lines, and Arcs.*

A Specimen *of the* Quadrant, *with its Apparatus, as mounted on a proper Pedestal, may be seen at Mr.* Dodsley's *in* Pall-Mall.

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DEFINITIONS.

A

ALmicanters, (ſo called by the Arabians) are circles of altitude, parallel to the horizon.

Amplitude, is an arc of the horizon in degrees and minutes, contained between the place of the riſing and ſetting of the ſun, moon or ſtars, and the eaſt and weſt points of the horizon.

Antartic Pole, is the ſouth and *Artic* the north pole of the world, which are diametrically oppoſite to each other.

Artic and *Antartic Circles*, are ſmall circles of the ſphere, diſtant from their poles 23 degrees 29 minutes; *Note*, a number of degrees and minutes is uſually wrote, as 23° 29′ is here marked.

Aſcenſional Difference, is the difference between the right and oblique aſcenſion or deſcenſion; in the ſun, it is the ſpace of time which he riſeth and ſetteth, before or after ſix o'clock.

Aſcenſion Oblique, is that degree and minute of the equinoctial, which riſes with the center of the ſun, moon, or ſtar, in an oblique ſphere.

Aſcenſion Right of the ſun, moon, or ſtar, is that degree of the equinoctial, accounted from the beginning of Aries, which riſes with it in a right ſphere, or ſuch a ſphere where the poles lie in the horizon.

Azimuths, or *Vertical Circles*, are great circles, interſecting each other in the Zenith and Nadir (as meridians or hour circles do in the poles) and cutting the horizon (as thoſe do the equinoctial) at right angles.

C

Circles of Longitude, are great circles of the ſphere, paſſing through a celeſtial object and the poles of the ecliptic, where they determine that object's longitude, reckoned from the beginning of Aries; and on theſe circles are the latitudes of ſuch objects meaſured.

Complement, is the filling up what any arc or angle wants of ninety degrees, or that part by which it exceeds ninety degrees, to make it up a hundred and eighty degrees.

Culminating, or *Culmen Cœli*, is the higheſt point in the heavens that any planet, or ſtar, can riſe to, in any latitude; and when a celeſtial object comes to the meridian of any place, it is ſaid to culminate: in north latitudes the ſouthing of the moon and ſtars is taken for the ſame thing.

D

Declination of the ſun, moon or ſtars, is their diſtance from the equinoctial, reckoned on a meridian, in degrees and minutes, and is either north or ſouth. The ſun's greateſt declination is 23° 29′.

Degree of a great circle of the ſphere is the 360th part thereof.

Deſcenſion of the heavenly bodies, is their going down, or ſetting, in the weſtern part of the horizon.

Deſcenſion oblique, is that part of the equinoctial, which ſets with the center of the ſun, moon, or ſtar, or with any point of the heavens, in an oblique ſphere.

E

Eclyptic is a great circle of the ſphere, interſecting the equinoctial in two oppoſite points, Aries, and Libra, making an angle therewith of 23 degrees 29 minutes, called the obliquity, of the ecliptic, equal to the ſun's greateſt declination: in this circle, according to appearance, is the ſun always found, and the earth truly in the oppoſite ſign, degree, and minute: it is divided into twelve equal parts, called ſigns, and every ſign into thirty degrees, every degree into ſixty minutes, and every minute into ſixty ſeconds; it alſo toucheth the two tropics in the beginning of Cancer and Capricorn.

Equinoctial in the heavens, or *Equator* on the earth, is a great circle of the ſphere, whoſe poles are the poles of the world: it divides the globe into two equal hemiſpheres, called north and ſouth.

Equinoxes are the preciſe times in which the ſun or earth enters into the firſt points of Aries and Libra, which happens about the ninth of *March*, and twelfth of *September*; which times are called the vernal and autumnal equinoxes, for then the days and nights are equal.

H

Hemiſphere is the half of a globe or ſphere, when it is ſuppoſed to be cut through the center in the plane of one of its great circles.

5 *Ho-*

Horizon is a great circle of the ſphere, which divides the heavens and the earth into two equal parts or hemiſpheres, diſtinguiſhed by the names of upper and lower: it is either a ſenſible or apparent, or a rational or true horizon. The ſenſible or viſible horizon, is that circle which limits our ſight, and may be conceived to be made by ſome great plane on the ſurface of the ſea. It determines the riſing and ſetting of the ſun, moon, and ſtars, in any particular latitude.

The rational, real, and true horizon, is a circle which encompaſſes the earth exactly in the middle, and whoſe poles are the Zenith and Nadir.

Hour Circles are the ſame with meridians or great circles, meeting in the poles of the world, and croſſing the equinoctial at right angles, they are drawn upon globes through every fifteen degrees of the equinoctial.

Hour is the twenty fourth part of a natural day, containing ſixty minutes, and each minute ſixty ſeconds. The aſtronomical hours, which always begin at the meridian, are reckoned from noon to noon.

L

Latitude Celeſtial is the diſtance of a ſtar or planet from the ecliptic, meaſured upon an arc of a circle of longitude, from the ecliptic towards the poles thereof.

Latitude on the earth is the height of the pole of the world above the horizon, which is always equal to the arc of the meridian, between the zenith and equinoctial.

Longitude celeſtial is the diſtance of a ſtar or planet, counted in the ecliptic from the beginning of Aries, according to the order of the ſigns, to the place where a circle of longitude paſſing thro' the object croſſes the ecliptic, ſo that it is the ſame as the ſtar's place.

Longitude, in geography, is an arc of the equator, intercepted between the firſt meridian and the meridian of the place; or it is the difference, either eaſt or weſt, between the meridians of any two places, counted on the equator.

M

Meridian (from Meridies) noon or mid-day, is a great circle of the ſphere, paſſing through both the poles of the world, and cutting the equator at right angles; unto which, when the ſun or any ſtar comes, it is the higheſt, or has then the greateſt altitude that it can have that day in that latitude. The ſtars are alſo ſaid to culminate, or be ſouth, when they are upon the meridian.

Meridian Angle is the angle made by the ecliptic and meridian at any given time of the day or night, which can never be more than ninety degrees when Cancer or Capricorn culminate, nor leſs than ſixty-ſix degrees thirty-one minutes, when Aries and Libra are on the meridian. It is of great uſe in the calculation of ſolar eclipſes.

Minute is the ſixtieth part of an hour in time, or of a degree in motion; an hour, or degree of a great circle, is ſub-divided into ſixty minutes, every minute into ſixty ſeconds, and each ſecond into ſixty thirds.

N

Nadir is the point in the heavens ſeemingly under the earth, diametrically oppoſite to the point directly over our heads, which is called the *Zenith*.

O

Obliquity of the ecliptic, is the angle that the ecliptic makes with the equinoctial at the firſt points of Aries and Libra, where it interſects therewith, and contains twenty-three degrees twenty-nine minutes, and is equal to the ſun's greateſt declination.

P

Poles of the World are two points, each ninety degrees diſtant from the equator; one ſituate to the north thereof, which is therefore called the north, or artic pole; and the other to the ſouth, which therefore is called the ſouth, or antartic pole.

Poles of the ecliptic are two points 23° 29′ diſtant from the poles of the world, lying exactly in the polar circles, and are each ninety degrees from the ecliptic.

Q

Quadrant is the quarter or fourth part of a circle; in this work it ſignifies an inſtrument of that figure which is graduated on the limb with ninety degrees. See its particular deſcription hereafter.

R

Radius is the ſemi-diameter of any circle, which being equal to the ſine of 90 degrees, is by ſome called the whole ſine.

S

Solſtice is the time when the ſun (apparently) enters the tropical points Cancer and Capricorn, and is got fartheſt from the equinoctial, where, before he returns back towards it, he ſeems to be for ſome time at a ſtand.

Southing of the ſtars is the ſame with the time of their culminating, or being upon the meridian; they have then juſt got half way of their journey, betwixt their riſing and ſetting.

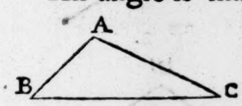
DEFINITIONS.

In the procefs of this treatife, fome marks, characters or figns will occur, which are explained as follows.

Characters.	What they fignify.
$+$	More, or addition.
$-$	Lefs, or fubftraction.
$\times$	Multiplication.
$\div$	Divifion.
$=$	Equality.
z	The fum.
x	The difference.
Si.	Sine.
Cofi.	Cofine, or fine of the complement.
Sec.	Secant
Cofec.	Co-fecant, or fecant of the complement.
Ta.	Tangent
Cota.	Co-tangent, or tangent of the complement.

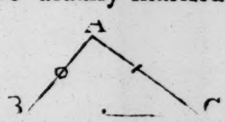
:, : :, are figns of proportion thus: fuppofe it to be, as 2 is to 4, fo is 8 to 16; that proportion is denoted thus, 2 : 4 : : 8 : 16.

An angle is marked $\angle$, and generally when reference is made to



an angle, as at B in the annexed triangle, it is denoted by $\angle$ A B C, or, if to the angle at C, by $\angle$ A C B.

If a fide or fides in a triangle are given, they are ufually marked or diftinguifhed by a fmall ftroke, as, fuppofe in the triangle annexed, A C and B C are given; then they are marked with fuch ftrokes acrofs them as appear therein; and when a fide is fought, it is marked with (°) as the fide A B is marked; and the like when the angles are given or required, as in the following triangle.



I N-

INTRODUCTION.

TRigonometry (for the eaſier and readier practice of which this Quadrant was deviſed) is that part of GEOMETRY that is employed in meaſuring triangles.

A plane triangle, the ſubject of plane trigonometry, conſiſts of ſix parts, *viz.* three ſides and three angles, any three of which being given, or known, the other three are readily found, except in the ſingle caſe of three angles, given without a ſide, in which caſe, the PROPORTION, not the MEASURE of the ſides is determined.

When two lines of a triangle meet in a point, the opening or diſtance between them, or, which is the ſame thing, the inclination of the one line to the other, is called an ANGLE, which when the lines forming it are ſtraight ones, is called a recti-lineal, or right-lined angle, as at A, fig. 1.

fig. 1.

But, if the lines forming the angle be crooked, it is then called a curvi-lineal angle, as that at B, fig. 2.

fig. 2.

And when one line is ſtraight and the other crooked, it is called a mixt angle, as at C, fig. 3.

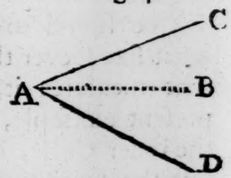
fig. 3.

The lines forming any angle are called its legs.

One angle is ſaid to be LESS than another, when its legs are more inclined, or nearer to one another; and, on the contrary, it is the bigger if leſs inclined.

Let

fig. 4.



Let there be two legs A D, and A C meeting in the point at A; if you imagine these lines to be moveable, like the legs of a pair of compasses, and fastened together at A as in a joint, it is easy then to conceive, that the farther they are opened, or parted from one another, the greater will be the angle between them; as, on the contrary, the nearer they are brought together, the angle between them will be so much the less; as in the figure above, the angle C A D formed by the two lines A C and A D, is greater than the angle B A C, formed by the lines A C and A B. But it must be noted, that the quantity of angles is by no means to be measured by the length of their legs A D and A C, or A B and A D, but by their inclination to one another, and by that only.

Every circle may be conceived to be divided into 360 parts or degrees, and every degree into 60 parts, which are called minutes; every minute also into 60 parts, which are called seconds; and every second into thirds, and so on.

And the reason why this number, 360, is made use of for the division of the circle, is, because it can be divided into a greater number of parts, without remainder, than any other number less than it.

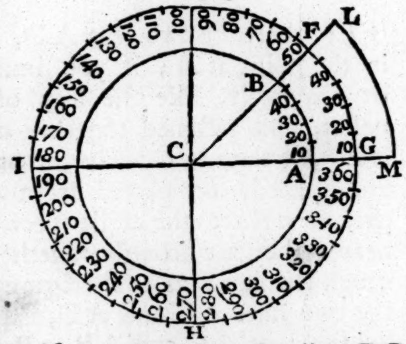
The mark usually made for a degree is ($^\circ$) for a minute ($'$) for a second ($''$) and for a third ($'''$): thus, forty-four degrees, seven minutes, fifty-two seconds, and twenty thirds, are thus exprest, $44^\circ, 7', 52'', 20'''$.

The measure of an angle is the arc of a circle, described on the angular point, and, therefore, the quantity of an angle, or the number of degrees it consists of, may be found by taking the angular point for a center, and thence drawing with the compasses a portion or part of a circle, to cut the legs that form the angle, and then by measuring the arc contained between them, by the method hereafter directed, the quantity of the angle will be determined.

On the [redacted] center C (see fig. 5) let there be formed two circles, an inner and an outer one; conceive the outer one to be divided from any point G, into 360 degrees; draw two strait lines F C, G C, forming at the angular point C, the angle FCG, and passing through or cutting the circumference of the outer circle; then, if the arc G F contains 45° in the outer circle, the same line F C will cut the inner circle (if divided into the same number of degrees) at 45° also;

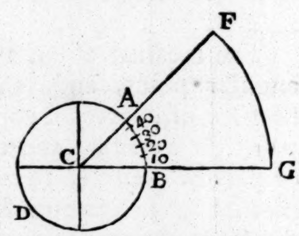
and

fig. 5.



and the quantity of the angle F C G will be found to be the same, of what size soever the circle be drawn that measures it, as here in the present example, the arc A B, in the inner circle, evidently contains just the same number of degrees as the arc G F does in the outer circle; and if you draw an arc beyond G F, suppose M L, this will still give the same measure of the angle L C M, as of the angle B C A; for, if you imagine the line F C to be carried round the central point C, according to the order of the letters F G H I, it is manifest, that the point F will describe the whole outer circle, in the same time that the point B describes the whole inner circle; or, if the line L C is carried on in any part of its round, *viz.* into the situation M C, the point F will have described just as great an arc of its circle, as the point L will have described of its circle; that is, the arcs F G, L M, and B A, shall each of them bear the same proportion to the circumference of its respective circle; so that if F G is an eighth part of the outward circumference, B A is an eighth part of the inner circumference; if one of these arches contain 45°, the other will likewise contain forty five degrees.

fig. 6.



It is in consequence of this proportion, that astronomers are able, with a small circle, a semi-circle, or Quadrant, to measure arches in those vast circles which we imagine in the heavens; for, conceive A B D in figure 6, to be a circle of brass, or other materials, whose circumference suppose to be divided into 360 parts; let F and G be two stars, whose distance from one another is to be measured; if the star G be viewed through two sights placed in the line C B, the eye being at C, and at the same time the star F be viewed through two other sights placed on a moveable ruler, whose edge coincides with the line C A, then will the number of degrees contained in the arc A B in the brazen circle, which on the figure is 45, shew the number of degrees in F G [illegible] arc of a circle imagined to be drawn in

in the heavens through the ſtars F and G, whence the diſtance of thoſe ſtars is found to be 45°.

It is uſual, in order to meaſure the degrees contained in an angle, to make uſe of an inſtrument called a PROTRACTOR, which is a ſemi-circle made of ſilver, braſs, or wood, divided into degrees; and, if it be large enough to be ſo divided, into halves and quarters of a degree.

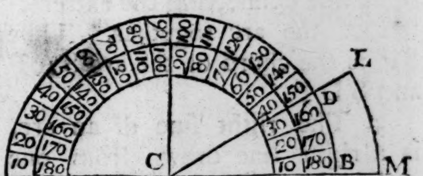


fig. 7.

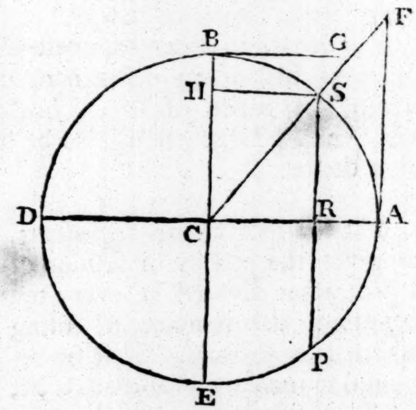
The ſeventh figure repreſents, or gives the picture or faſhion of a protractor divided at every ten degrees: the manner of uſing which is as follows. If it be required to meaſure an angle L C M, lay the central point of the protractor upon C, the angular point; and the ſemi-diameter C B upon C M, one of the legs of the angle to be meaſured; then the arc of the protractor D B, contained between the legs of the angle, ſhews the number of degrees contained in the angle L C M, which in the preſent caſe is 30°.

By the ſame inſtrument may an angle be drawn, containing any number of degrees required; ſuppoſe 30, draw a ſtrait line at pleaſure; ſuppoſe C M, lay upon it the ſemi-diameter of the protractor C B, ſo that its central point may fall upon C, that point of the line at which the angle is to be drawn; then make a mark at D, the diviſion of the protractor anſwering to 30°, with a fine pen or pencil, cloſe to the circumference thereof; then take off the protractor, and draw a line from C through D, *viz.* the line C D L, ſo will the angle L C M contain 30°, which was required.

Since the exact proportion of the radius, or ſemi-diameter of a circle, to the circumference thereof, is not to be expreſſed perfectly by numbers, Mathematicians have invented and applied to the circle, ſuch lines as are proper to ſupply that defect; ſuch as chords, ſines, tangents and ſecants, commonly made uſe of in Trigonometry, all which are graduated on the Quadrant hereafter deſcribed. The nature and conſtruction of which, ſo far as is neceſſary, on the preſent occaſion, to prepare the reader for practice, may be explained as follows.

1ſt. The radius of a circle is a right line drawn from the centre to the circumference.

fig. 8.



Thus (in fig. 8.) C A is the radius of the circle A B D E, and is equal to half the diameter A D; and the lines CS, CB, CD, CE, and all lines that can be ſo drawn, are each ſeverally a radius.

2. The chord of an arc, is a right line connecting the extremities of the arc together. Thus SP is the chord of the arc S A P and S D P.

3. The right ſine of an arc, is a right line drawn from one end, or termination of an arc, perpendicular to a radius, which is drawn to the other end or termination of the arc; thus S R is the right ſine of the arcs SA and S D.

4. The verſed ſine of an arc, is that part of the radius which is contained between the right ſine and the arc. Thus R A is the verſed ſine of the arc SA, and R D is the verſed ſine of the arcs S D.

5. The tangent of the arc SA, is a right line, FA, drawn without the circle, perpendicular to a radius C A, paſſing thro' A, one end of that arc, touching the circumference at that point A, and meeting the ſecant of the ſame arc in the point F.

6. The ſecant of the arc SA, is a right line C F, drawn from the centre C, thro' S, the other end of the arc SA, and meeting the Tangent FA in the point F; thus C F is the ſecant of the arc SA.

7. The complement of an arc is ſo much as it wants of a quarter of a circle. Thus the arc S B is the complement of the arc S A; the ſupplement of an arc is ſo much as it wants of a ſemi-circle; thus the arc S D is the ſupplement of the arc SA; and every ſupplement of an arc hath the ſame right ſine, tangent, and ſecant as the arc itſelf.

But ſince S H, BG, and C G, are, ſeverally, the right ſine, tangent, and ſecant of the arc S B, therefore they are, ſeverally, the right ſine, tangent, and ſecant of the complement of the arc S A, and, for brevity's ſake, are moſt commonly called the co-ſine, co-tangent, and co-ſecant thereof.

8. The verſed ſine R D of an arc S D, greater than a Quadrant, is greater than the radius C D; but R A, the verſed ſine of the arc SA, leſs than a Quadrant, is leſs than the radius.

9. And

9. And (becaufe CR, the part of the diameter contained between the right fine SR and the centre C, is equal to SH, the cofine or fine complement of an arc SA) it is evident, that the fum, or difference of the radius, and the cofine of an arc, will give the verfed fine of that arc, for DC, more CR, is equal to DR, the verfed fine of DS; and CA, lefs CR, is equal to RA, the verfed fine of AS. *1ſt Hodgſon, 56.*

10. Now, for the more eafy calculating of the proportions arifing amongft thefe fines, tangents, and fecants, let it be obferved, that there is a fpecies of numbers, called Logarithms, contrived at firft by Lord *Neper*, and adapted to the fines, tangents, and fecants of every degree and minute of the Quadrant; by which logarithms, all calculations are moft readily and commodioufly performed; and fcales of thefe logarithms have been made and fet on divers inftruments, for the more expeditious folution of all trigonometrical queftions and problems.

And, as thefe logarithmetic fcales are applied to, and put on, the prefent Quadrant, thefe problems may be folved thereby, without having recourfe to the logarithmetic tables, or any other inftrument.

11. The whole art and fcience of Trigonometry, is contained in this one propofition, *viz.* From three conftituent parts of a triangle, to find the reft: hence the various queftions arifing from changing the things given and required, are called the feveral cafes of Trigonometry; and in plane Trigonometry, they are ufually reduced to thirteen, *viz.* to feven in right, and fix in oblique angled plane triangles. *1ſt Hodgſon, 102.*

To render thefe as eafy as poffible for practical operation, it will be proper to premife,

1. That any two fides of a plane triangle, taken together, are greater than the remaining third fide.

2. That the greateft fide of every triangle is oppofite to the greateft angle, and, converfely, the greateft angle is oppofite to the greateft fide.

3. That the fum of the angles of every plane triangle, is equal to a femi-circle, or 180°.

4. Wherefore it follows, that if any two angles of a plane triangle are known, the third is known alfo, being found by fubtracting their fum from 180 degrees.

5. If one angle be obtuſe, that is, greater than a Quadrant, or ninety degrees, each of the other two will be acute, that is, each of them will be leſs than ninety degrees.

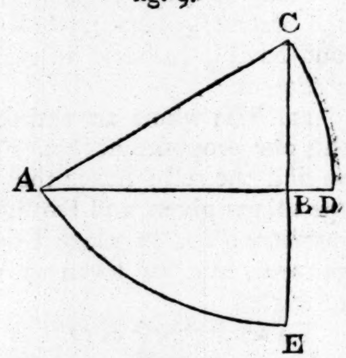
6. If one angle of a triangle be right, or ninety degrees, the other two, together, will be equal to that one right angle, or ninety degrees.

7. Wherefore in a right angled plane triangle, if one of the acute angles is given, the other is alſo known, being found by taking the given angle from ninety degrees.

8. In every right angled plane triangle, when the hypothenuſe, or longeſt ſide is made the radius of a circle, the other legs or ſides will be the ſines of the oppoſite angles.

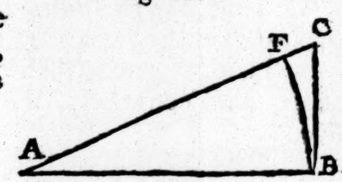
But, if either of the ſides, or legs, containing the right angle, be made the radius, the other ſide or leg will be the tangent of its oppoſite angle, and the hypothenuſe will be the ſecant of the ſame angle.

fig. 9.



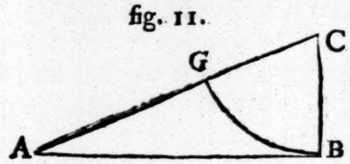
Thus in the triangle, fig. 9. If with AC, the hypothenuſe, as radius, the arc CD be deſcribed, then will BC be the ſine of the arc CD, the meaſure of the angle at A; and if one foot of the compaſſes be applied to C, and the arc AE be deſcribed, then will AB be the ſine of the arc AE, the meaſure of the angle at C.

fig. 10.



Again (in fig. 10.) AB being made the radius, BC is the tangent of the arc BF, the meaſure of the angle A, and AC is the ſecant of the ſame arc.

fig. 11.



Alſo (in fig. 11.) BC being made radius, AB is the tangent of the arc BG, the meaſure of the angle C, and CA is the ſecant of the ſame arc.

From

2

From what has been ſaid, may be inferred the following rule.

That, in the ſolution of a right angled plane triangle, if a ſide is required, any ſide may be made radius; but if an angle be required, one of the given ſides muſt be made radius. Alſo, if a ſide is required, begin the operation with an angle; but if an angle is required, begin with a ſide. Rule.

Before I conclude this Introduction, I muſt farther take notice, that the tranſpoſitions of the mean and extreme terms of four proportional numbers, or quantities, are frequently made uſe of hereafter, which therefore ſhould receive ſome explanation.

To this end obſerve, that four quantities, or four numbers, are ſaid to be proportional, when the two mean, or middle terms, multiplied together, are equal to the two extreme terms, multiplied by each other. Wherefore, when two products are equal to one another, we may conſider the two quantities, or numbers, which belong to one product, as the extremes of that proportion, and *vice verſa*.

Suppoſe for inſtance, the proportion to be, as two is to four, ſo is eight to ſixteen; ſince two, which is one extreme, multiplied by ſixteen, which is the other extreme, is equal to the product of the middle terms, four multiplied by eight, we may draw theſe inferences; as two is to four, ſo is eight to ſixteen: it will hold alſo, as two is to eight, ſo is four to ſixteen; or making two and ſixteen, which before we conſider'd as the two extremes, to be the two mean terms, and four and eight the two extremes, it will be, as four is to two, ſo is ſixteen to eight; or, as eight to two, ſo is ſixteen to four.

The terms of four proportionals may be changed in various other orders and manners, of which inſtances are given in many authors, by inverting, alternating, compounding, and dividing them; but theſe are ſufficient for the preſent purpoſe, and ſo I paſs on to the deſcription of the Quadrant, and its apparatus.

DE-

DESCRIPTION
OF THE
QUADRANT,
AND
Its APPARATUS.

THE Quadrant, when it is intended to take an altitude, is mounted, or hung vertically on a pedeſtal, by an axis having a male ſcrew affix'd to the back of it, and a female ſcrew to fit it; and upon this the Quadrant moves upwards or downwards, and the movement may be eaſed, ſtiffened, or faſtened as occaſion requires. Or, the Quadrant may, if occaſion require, be taken off its ſcrews, and be placed at the top of the pillar or column, and move horizontally; and then, if a moveable ruler, with two ſights more than what it uſually has, were properly placed on it, it would ſerve for ſurveying and meaſuring diſtances or lands. But the ſights which are uſually put on a Quadrant, are deſigned for taking of altitudes, and will be the only ſights taken notice of in this treatiſe.

In the centre there is placed a thread ſomewhat longer than the radius of the Quadrant, with a plumbet to it; which is not only uſeful for taking altitudes, and marking out their degrees, but may alſo be made uſe of with a pair of compaſſes, almoſt upon all occaſions, in practice, as hereafter will be ſhewn at large.

The centre where the thread is put through, is conſidered, in this treatiſe, as the uppermoſt part of the Quadrant.

The

The pedeſtal has a round bottom or foot, in which ſcrews are placed, in order to ſet the Quadrant level.

The figure of the Quadrant, with its lines and arcs, is printed off, and annext to this treatiſe; and you will obſerve, thereon, two marks on the vacant parts of the line of Tangents, which are deſigned for the places, where two braſs or ſilver grooves are to be fixed; upon which the aforeſaid two ſights, to be made alſo of braſs or ſilver, are to be put when altitudes are taken, and laid aſide when that purpoſe is anſwered.

The print (to make it ſerviceable for common practice) muſt be neatly paſted on a ſmooth board, framed, ſized, and fitted to it; and there ſhould be a looſe handle made to be put on, for the eaſier holding and managing the Quadrant, when it is to be uſed in common practice with the compaſſes and thread; and to be taken off when altitudes are to be obſerved.

It remains only to add upon this occaſion, that, in order to ſave the Quadrant from being ſcratcht with the compaſs points, it may be proper to put little braſs pinholes, in the places moſtly uſed; ſuch as the radius or 90 degrees on the ſines; 45 degrees on the tangents; 23° 29′, the ſun's greateſt declination; and 66° 31′, the complement of it, on the line of ſines; as alſo, on 10 on the equal parts; and 180° on the verſed ſines: all, or any part of which work, as alſo of the other works, neceſſary for the mounting, or fitting the Quadrant for its uſes, may be eaſily performed by any mathematical inſtrument-maker, if the reader chooſes it.

Deſcription of the lines and arcs on the Quadrant, with an account of ſome of the inſpectional uſes of it.

In the left hand edge of the Quadrant, from the centre downwards towards the limb, there is placed a line of equal parts numbered; 1, 2, 3 *&c.* to 10. At 10, where this line of equal parts ends in Mr. Collins's Quadrant, the ſame touches the line of ſines in the limb of Mr. Collins's Quadrant; but in this of Mr. Rowley's, it makes an opening or angle of 3 degrees and 20 minutes; ſo that an arc taken from the end of the line of equal parts on the line of ſines, which is placed at the limb, will be 3 degrees and 20 minutes too much, *ex. gr.* if the arc propoſed to be taken is 20 degrees, and a pair of compaſſes be extended on the limb, from the end of the line of equal parts to the figures 20, too much will be taken by 3 degrees and 20 minutes, becauſe it takes in the additional ſpace of 3° and 20 minutes, between the equal parts and the beginning

Equal parts.

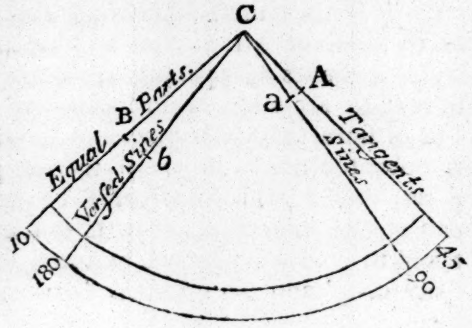
ning of the ſines on the limb; and therefore the compaſſes ſhould be extended no farther than 16 degrees and 40 minutes, for then the arc contained between the points of the compaſſes will be juſt 20 degrees, as required. And the ſame caution muſt be uſed in the line of tangents. But this, however, may be avoided, in the manner hereafter mentioned.

Verſed ſines. Upon the ſame left hand edge, adjacent to the line of equal parts or numbers, there iſſues from the centre towards the limb, a line of ſines, ending at 90°, and a line of verſed ſines, whoſe radius is equal to half the radius of the Quadrant, beginning at 90°, where the line of ſines ends, and extending to 180°.

Line of tangents. On the right hand edge of the Quadrant, there runs out from the centre, towards the limb, a line of tangents, graduated to a radius, equal to the radius of the Quadrant.

Line of ſines. On the ſame right hand edge with the tangents, there runs out from the centre towards the limb, a line of ſines to the radius of the Quadrant.

But obſerve, as before, this line of tangents does not join, or fall in cloſe with the line of ſines, no more than that of the equal parts does with the verſed ſines. If, on the contrary, both theſe lines had run cloſe to one another, there would then have been no need of making an allowance for the before mentioned 3° 20′, in either caſe. I am apt to think, that the ſeparation of the lines here mentioned, was made for the purpoſe of introducing the two ſights into the vacancy, ſo as to run parallel with the edge of the Quadrant. But we may very eaſily remedy this, by tranſpoſing, or transferring, the foot of the compaſſes from either the line of equal parts, or that of the tangents, to the reſpective neighbouring or adjacent lines of verſed ſines and common ſines, directly croſs from one to the other; and this will have the ſame effect with the other method of making an allowance for the 3° 20′, and with leſs trouble.



Suppoſe, for inſtance, in the adjacent figure, repreſenting the Quadrant, one foot of the compaſſes, in common practice, was applied to C the centre, and the other extended to A, on the tangents; inſtead of this, ſet or bring it down by the eye to *a* on the ſines, and the 3° 20′ are then regained;

gained; ſo likewiſe, if it be ſet at B on the equal parts, bring the foot of the compaſſes down to *b* in the neighbouring ſines, and the defect is ſupplied; ſo alſo, if it were to be placed at (10) on the equal parts, you may bring the foot of the compaſſes to 180 degrees on the verſed ſines; or, if at 45° on the tangents, you may carry it to 90° on the ſines: of all which there will be ſeveral examples in the progreſs of this tract; and in many caſes where radius is one of the terms of a proportion, and where the laſt term is taken on the tangents, if the leg of the compaſſes is applied to radius, on the ſines, inſtead of the tangents, the 3° 20′ is regained.

As the line of tangents on Mr. Rowley's Quadrant goes to 45° only, therefore, if in any proportion given there occurs one or more tangents exceeding 45°, that proportion may be reſolved into more proportions than one, by a ſubſtitution of their co-tangents, upon the maxims of, and in the manner deſcribed by, Mr. Collins page 72, 73, 148, 149, and 169, or as directed hereafter in thoſe caſes.

Near the centre of the Quadrant ſtands a table of four columns, marked over head, Months, Firſt Year, Third Year, Leap Year. The Table.

In the firſt column are the names of the months, oppoſite to each of which ſtand three numbers, repreſenting minutes, in each column, which minutes are to be added to, or ſubtracted from, the ſun's declination found by the Quadrant, according as they ſtand noted by the letters A or S, of which more hereafter.

The firſt number is for the firſt ten days, or beginning of the month; the ſecond number is for the middle, and the third for the latter end of the month; whence, by uſing the number in the firſt, third, or fourth column, as hereafter mentioned, the ſun's declination found by the Quadrant (which is fixed only to the ſecond year after leap year) may be corrected, and made to ſerve for the firſt or third years, or for leap year itſelf. And this is what is called by Mr. Collins, at the end of his book, the rectifying table.

Underneath the rectifying table are four quadrantal annuli; the loweſt contains the repreſentation of certain fixed ſtars, oppoſite to each of which are contained, in the next annulus, their names; in the third, their declinations; and, in the fourth is marked, whether the declination is north or ſouth. There ſhould have been likewiſe, a mark of diſtinction, ſuch as + (or more) to ſuch ſtars as have more than twelve hours right aſcenſion, *i. e.* ſuch as riſe after the autumnal equinox, or firſt degree of Libra, as is done by Mr. Collins, and therefore I have added it to the print. The annuli of the ſtars.

 Below

Below the laſt mentioned annuli are two lines, called Quadrants of right aſcenſion, numbered from the left to the right, the higheſt 6, 7, 8, 9, *&c.* the loweſt 1, 2, 3, 4, *&c.* the thread which ſuſtains the plumbet being laid over any ſtar, in the annulus immediately above, cuts the lower line at the hour of the ſtar's right aſcenſion, when the ſtar riſeth after the preceding equinoctial point; but the higher line is cut by it, at the hour of its right aſcenſion, when the ſtar riſeth after the preceding ſolſtitial point; but which of the two, the equinoctial or ſolſtitial points, is the preceding, cannot be known by this Quadrant only, but may very eaſily be done, by viewing the cœleſtial globe, and ſeeing there after which of the two points the ſtar riſeth; but as to theſe two annuli, and their uſes, the ſame will be explained more particularly when we come to ſolve ſuch problems as relate to the ſtars.

Lines of ſeaſons. Below the laſt mentioned annuli are (contained in four lines) the four ſeaſons of the year, divided into months and days, beginning with the ſpring months in the loweſt line, to be reckoned from the left to the right hand, and continued through the ſummer in the line immediately above it, from the right hand towards the left, *&c.*

The firſt of January, and every ſeventh day following through the year, is diſtinguiſhed by ſmall dots, for the more ready finding what day of the week any day of the month falls on.

Next below theſe is a quadrantal line of the ecliptic, and immediately below that, a line of the ſun's declination.

Secants. Following the above is a line of ſecants, to a radius, equal to half the radius of its Quadrant, equal to the diſtance between ten and the end of the line of hours, at the right hand of the ſecants, and muſt, when this is uſed as a radius, be entered twice down the line of ſines from the centre.

Verſed ſines of ſome hours. Here note, that the above line of hours, is part of a line of verſed ſines, and is numbered with hours and minutes inſtead of degrees, to ſerve for finding the hour from noon more exactly than can be done by the other lines; this is called by Mr. Collins the quadrupled verſed ſines. Below this is a Quadrant of a circle, containing a line marked verſed ſines two radius's, or a radius doubled, equal to the radius of its Quadrant, *i. e.* the ſingle radius meaſured from ſixty to 0, (beyond 10, on the right hand) doubled, is equal to the radius of its Quadrant; ſo that when an entrance is neceſſary, as hereafter it will be found to be, it muſt be made twice down the line of ſines, from the centre to this annulus.

Under

Under the above line of verſed ſines, is a line of hours fitted to it, and their uſes are explained in Mr. Collins (from page 181 to 190.) and in part hereafter. But note, that the line of verſed ſines, uſed by Mr. Collins, is a ſingle line, whoſe radius is equal to half the radius of the Quadrant. Line of hours.

Wherefore, whenever he mentions a ſingle entrance from the centre down the line of ſines, to apply his rule to this double line, the entrance from the centre muſt be doubled, as it is quadrupled, in uſing his quadrupled verſed ſines.

Below this is a quadrantal line of ſines, which iſſues from the edge to 10, 20, *&c.* and ſo to 90°, each degree being ſub-divided into ſixty minutes, reckoning each ſtroke (ſix in number, including the ſtroke of the next degree) ten minutes. In reading of this line, obſerve that from the left to the right edge, the number 10, 20, *&c.* are in large characters; but, in the return of the line from the right to the left, it is in ſmaller characters.

Underneath this line of ſines there is a line of hours, both which lines may be variouſly uſed, as occaſion requires, and in ſome particulars, as hereafter.

And now the only lines remaining to be explained, are, a ſtrait line of verſed ſines, with an hour line fitted to it, and two circular arches of months underneath. Theſe lines are peculiar to this Quadrant. The line of ſines is placed at a diſtance from the centre, equal to the latitude of London, meaſuring it from the centre of the Quadrant to 90 over the hour ſix; for, on applying that meaſure to the line of ſines, it reaches to 51° 32′. Verſed ſines and hour line.

The thread being laid over the verſed ſine of 90, or at VI, in this right line, cuts that right line at right angles, and the limb of the Quadrant at 60, from the left edge.

Thus far as to the deſcription of the lines and arches on the Quadrant.

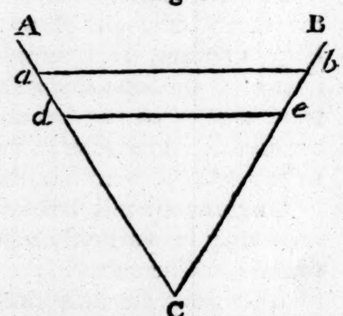
It now remains, before we proceed to treat of its uſes, to note, that they ſubſiſt upon the ſame principles as in the Sector; to evidence which, let us compare it with the Sector, ſo far as is neceſſary for the preſent purpoſe.

The Sector, as it is geometrically defined, is a figure bounded by two right lines, and part of the circumference of a circle; but, by a Sector, here ſpoken of, we are to underſtand, an inſtrument conſiſting of two legs, that open upon a centre or joint, like a carpenter's ruler. The lines of equal parts on ſuch a Sector, as well as that on the left ſide of Mr. Rowley's Quadrant, are divided into an

hundred ſuch parts, and, if the length of the inſtrument permits, again ſub-divided into halves and quarters.

Theſe diviſions are placed on each leg of the Sector, and are numbered 1, 2, 3, 4, *&c.* to 10. but, in this Quadrant there is but one of theſe lines, and this will make a difference between the working the ſame problems by theſe two inſtruments, of which hereafter. Here noting, that 1 may be taken, both in the Sector and Quadrant, for 10, 100, or 1000, as occaſion requires; and then, 2 will ſignify 20, 200, 2000, *&c.*

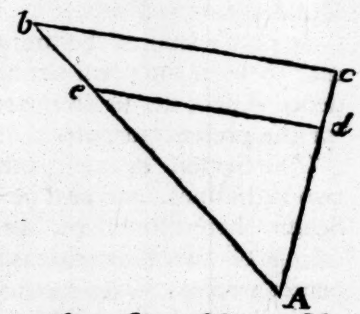
fig. 1.



The invention and contrivance of this inſtrument, called the Sector, no doubt aroſe from a conſideration of the fourth and fifth propoſitions of the ſixth book of Euclid, which demonſtrate, that ſimilar triangles have their ſides proportional. For, let the lines CA, CB, fig. 1. repreſent the legs of the Sector, and let C *a* and C *b* be two equal ſections from the centre, and C *e* and C *d* two other equal ſections from it. Then, if the points *a* and *b*, and the points *e* and *d* are ſeverally joined by two right lines, they will be parallel by the ſecond propoſition of the ſixth book of Euclid. And if the lines *a b* and *d e* are parallel, the triangles C *a b* and C *d e* will be equi-angled (by the ſcholium of the fourth propoſition of the ſixth book of Euclid) and, therefore, by the ſaid propoſition the ſides C *e* and C *b*, C *d* and C *a* will be proportional; that is, as C *e* is to C *b*, ſo is C *d* to C *a*; and the like reaſon holds in all other the like ſections.

Barrow's Euclid, 116.

fig. 2.



The lines on the Sector are diſtinguiſhed into two ſorts, *viz.* lateral and parallel; lateral are ſuch as are found on the ſides of the Sector, as C *e* and C *b* (in fig. 1.) and A *d* and A *c* (in fig. 2.); parallels are the lines that run from one leg of the Sector croſs over to the other; and, therefore, a lateral entrance is the proportion taken from the centre to any part of the ſide of the Sector, and the parallel entrance is when taken from ſide to ſide.

 When

When you open the legs of the Sector, they keep the transverse numbers parallel to themselves, so that to whatever degree the Sector is opened, yet the numbers on the line of lines, or equal parts, are parallel; the number nine, for instance, on one leg, being still opposite to nine on the other, 8 to 8, 7 to 7, *&c.*

But this is not the case of the Quadrant, for here is but one leg or line, and that fixed or immoveable; and, though the thread is introduced to supply the place of the other leg in the Sector, yet the same moving only by itself, departs from, and leaves its line of equal parts stationary, nor can it receive, nor is it made to receive, the like numbers impress'd upon it, as those are that are on the fixed leg. Hence it is, that when the thread is made use of in solving any problem, as it is carried from the fixed line of equal parts, it gradually declines from a parallelism with it; so that the thread would be useless, if something else had not been devised, to answer the like purposes and uses with those of the Sector.

Now, as taken notice of above, with respect to the proportionality of similar triangles, as demonstrating the uses of the Sector; so, upon the same principles, the thread, and the directions for the uses of it on the Quadrant, are founded: for, in what proportion, or to what degree soever, the thread is moved, within the limits of the Quadrant, it forms, with the line of equal parts and the arc at the limb of the Quadrant, a triangle; and, if at any one number on the line of equal parts, suppose the number five, one foot of the compass is set, and the other carried out so, as just to touch the thread, and thence to pass off without cutting it, it then forms a right angle at the point of contact, the lines from A *d* and *d c* (in fig. 2.) being tangents of the small invisible arcs, made by the points of the compasses in that operation.

Now since in each of the triangles, A *d e*, and A *c b*, the angles at *d* and *c* are right ones, and the angle at A common to both, consequently the angles at *e* and *b* are equal, and, therefore, these triangles are proportional, as well as those on the Sector, and so will answer alike to all uses and purposes.

For example, suppose you was to multiply 8 by 6; then, to perform this Sector-wise, take 8 on the leg of the Sector, from the centre in the compasses, and set this extent over from 10 to 10, at the end of the Sector; then take the parallel distance between 6 and 6, on the line of lines in the compasses, and apply one foot of the compasses at this extent to the centre, and then the other, turned round, will reach

Brown on the Quadrant, 78, 79, 80.

reach to 4,8, or rather $4\frac{8}{10}$ laterally. Or ſhorter, thus, as the parallel 10 to the lateral 8, ſo the parallel 6 to the lateral 4,8, or $4\frac{8}{10}$.

Now, to perform the ſame by the Quadrant, ſet one foot of the compaſſes at the centre of the line of equal parts, and extend the other to 6 on the ſame line, and, preſerving this extent in the compaſſes, apply one foot to 10, at the end of the line of equal parts, and move the other foot ſide-ways, and the thread towards it, till they juſt meet and touch, but do not cut each other; *(and this kind of operation we will for the future call an entrance of the compaſſes.)* Then, the thread remaining in the ſame poſition, take the neareſt diſtance between 8 and the thread; that is, ſet one point of the compaſſes on 8, and let the other juſt touch the thread without cutting it; and this extent, applied to the centre, will reach to 4,8, or $4\frac{8}{10}$, as before.

Or thus: take 8, laterally, on the line of equal parts, and at that extent enter one foot of the compaſſes at 10, and bring the thread to the other foot, as before; then, without moving the thread, apply one foot of the compaſſes (diſcharged of the former extent) to 6 on the ſame line, and thence take the neareſt diſtance to the thread; then that diſtance will reach from the centre to 4,8, or $4\frac{8}{10}$; and thus the proportion is, as 10 to 8, ſo 6 to 4,8, or $4\frac{8}{10}$.

To perform diviſion of natural numbers by the Sector (as ſuppoſe to divide 40 by 5.) the rule is, as the lateral 5 to the parallel 10, ſo is the lateral 4, eſtimated as tens, or 40, to the parallel 8. Take 5 on the Sector, laterally, in the compaſſes, and (at that extent) apply one foot of it to 10, and, carrying out the other foot parallely, bring the number 10, on the other leg of the Sector, to it. Then take the lateral diſtance from the centre to 4 in the compaſſes, and with that extent draw down the compaſſes between the two legs, and they will reſt at 8, the quotient.

The ſame caſe upon the Quadrant; take the diſtance between the centre of the equal parts and 5, in the compaſſes, enter one foot at 10, and bring the thread to meet or touch the other foot, and keep the thread in this poſition; then ſet one foot of the compaſſes (diſcharged of the firſt extent) at the centre, and extend it to 4 on the equal parts, and drawing the compaſſes at that extent down between the line of equal parts and the thread perpendicularly, as near as you can by the eye, to the thread, and you will find that one foot of them will reſt at 8, on the equal parts, the quotient as before.

Thus you ſee, that the operation by the line of lines on the Sector, and by the equal parts on the Quadrant, agrees; the opening or

removing the thread, from the line of equal parts, on the Quadrant, is the ſame with the opening the legs of the Sector, and the taking the neareſt diſtance from any point or number on the equal parts, to the thread, on the Quadrant, is the ſame as taking it, parallely, from point to point, or from number to number on the Sector.

And now I muſt obſerve, once for all, that when it is ſaid, lay the thread to the other foot of the compaſſes, it is meant to lay the thread perpendicular to the line joining the two feet or points of the compaſſes, ſo that the ſaid foot, when turned about, may juſt touch, and not cut over the thread; and when I ſay, draw down the compaſſes, between the ſcale and the thread, the words, *perpendicularly to the thread*, are to be underſtood; and the ſame thing is meant when I ſay, enter them between the thread and the *Scale*. And, when any length is taken out of the ſcale, from the centre, it is ſaid to be lateral, or a lateral entrance; and when taken tranſverſely, or from the ſcale to the ſtring, it is ſaid to be parallel, or a parallel entrance.

And thus having cleared the way to the uſes of the Quadrant, let us now proceed to explain or ſhew the ſame, under theſe heads.

1. The methods of taking altitudes by the Quadrant.

2. Such uſes as appear by, or are deduced from, a bare inſpection of the projection on the Quadrant, with the help or application only of the thread.

3. The uſe of the Quadrant, in ſolving ſome of the moſt common problems in Navigation, depending upon the rules of plane Trigonometry.

4. The uſe of the Quadrant, in ſolving ſome problems in Aſtronomy, depending as well upon right angled as oblique angled ſpherical triangles.

1. And, firſt, as to the method of taking altitudes by the Quadrant. Bring the foot of the pedeſtal to a true level by its ſcrews, ſo that the ſtring of the Quadrant may barely touch, and not bind upon the Quadrant.

Move the Quadrant round till you find it point to the Quarter, or place where the object is.

Then elevate the edge of the Quadrant, till you can ſee the object through both ſights; and this being done, the ſtring with its plumbet, will mark out, on the limb of the Quadrant, the degrees and minutes of the height of the object. But becauſe the looking through the ſights at the ſun, if that be the object, hurts the eye, you may prevent it, by bringing the object as near as you can, by gueſs, to the edge of the Quadrant, and then hold a piece of white paper under the ſights, and move the Quadrant, as the image of the ſun ſeen on the

the paper, guides you, and you will, by degrees, have the sun's image passing through the centre of the cross hairs, in the uppermost sight, and then you will find that the image, which cuts the centre of the cross hairs, passes also through the little hole, in the same sight, down to the two little holes on the sight below, and then the altitude of the sun is fixed, and the string at the limb gives the degrees and minutes of his altitude.

But, as to other objects, the taking the altitude of which cannot offend the eyes, there is no occasion in these cases for the caution above, but their heights are taken in the common way.

2. The next thing in course, is, to shew the use of the Quadrant in solving some problems, by a bare inspection, with the application, only, of the thread.

Declination. And, in the first place, let it be required to find, upon a given day, the sun's DECLINATION.

In order to this, observe, that the line of declination on the Quadrant, is so divided, that each degree is parted from the other by long strokes, containing five short ones between them, each of which is made for ten minutes, and (with one of the long strokes) make together one degree, or sixty minutes, and, consequently, the middlemost in each division is made for, and denotes, 30 minutes.

Observe also, that (towards the end of the line of declination) is 23 degrees, and next to that is 25, which is not 25 degrees, but 25 minutes, for the whole declination is but 23 degrees and 29 minutes.

These things being premised, let the question be,

PROBLEM I.

Problem 1st. What is the sun's declination on the 27th day of April, the second year after leap year, to which year the Quadrant is fitted?

Lay the thread over the day of the month, in the upper circular arcs of months, and it will cut the line of declination in 17 degrees and 7 minutes, which is the declination sought.

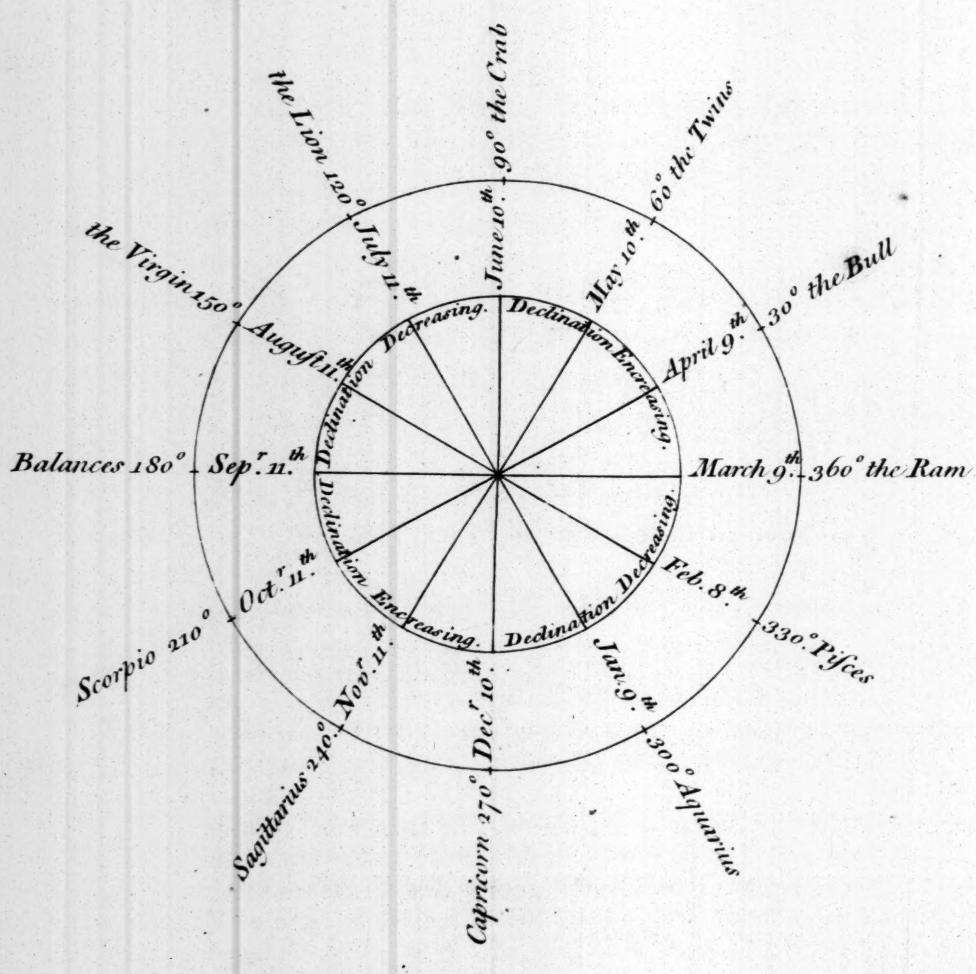
But for the other years, *viz.* the first and third years after leap year, and for leap year itself, you must add to, or subtract from, the declination, as is directed by the rectifying table.

The declination may be found also, if the right ascension is given: thus, suppose the right ascension to be 55° 17′, lay the thread over 55° 17′ on the limb of the Quadrant, from the left to the right, and it will cut 19° 39′ on the line of declination.

Or, if the sun's place is given, as suppose in Taurus 27° 33′, lay the thread over that place, and it will likewise cut the line of declination at 19° 39′.

Or

To face page 25.



Or, if the time of the ſun's riſing be given, as ſuppoſe 4 hours 10 minutes, lay the thread on the ſtrait line of hours, according to that time, and you will find it cuts the 10th of May, in the lower arcs of months, and then laying the thread to that day, on the upper arcs of months, it cuts the line of declination at 20° 10′.

PROBLEM II.

To find the ſun's place by Inſpection.

Suppoſe his place is required on the 27th day of April, lay the thread, as before, on that day on the upper arcs of months, and it ſhews the ſun's place in the ecliptic, on all the ſeveral days of the month it covers. Problem 2d, ſun's place.

For example, in the preſent caſe, it lieth on the 27th day of April, * the 25th day of July, the 30th day of October, and the 21ſt day of January nearly. And the thread thus placed falls between the ſigns of Taurus and Leo for the two ſummer months, *viz.* April and July, and between Aquarius and Scorpio for the two winter months October and January. Now, to know which of theſe ſigns is the true one, look on the ſcheme annexed, for the day of the month on which the ſun enters either of them, and this will guide you to the right ſign; and the thread, as laid over the day of the month, will ſhew you, on the ecliptic, the degree and minute the ſun is in that ſign.

Thus, by the annex'd ſcheme, it appears, that the ſun enters the Bull, or Taurus, the 9th of April; wherefore the thread lying on the 27th of April, the given day, cuts the Quadrant in Taurus at 17° 07′.

July the 25th the ſun (by the ſcheme annex'd) is in Leo, and the thread cuts the ecliptic at 12° 53′, therein.

October the 30th the ſun is in Scorpio, and the thread cuts at 17° 07′, therein.

January the 21ſt the ſun is in Aquarius, and the thread cuts at 12° 53′, therein.

Note, if the ſun's place is required, and any day of the month is given, you may find it (not only by the above method) but alſo if the ſtring is laid over his preſent declination; it gives the required anſwer: or, if it is laid over his right aſcenſion, in the limb of the Quadrant, it does the ſame, and ſo *vice verſa.*

But (becauſe two of the ſines (*viz.* ♈ ♎) are put together on the left edge of the Quadrant, and then ♓ ♍ ♉ ♏, and after them ♒ ♌ ♊ ♐, are put together; and, laſtly, ♑ ♋ on the right edge of the

E Quadrant)

* All the days of the month mentioned throughout this tract, are adapted to the old ſtile.

Quadrant) a difficulty may ariſe from the placing the ſigns, *viz.* how to know which is the ſign required; in order to obviate which, the foregoing ſcheme, ſhewing the entrance of the ſun into theſe ſigns, was thought to be of uſe; and, for the further clearing and removing this difficulty, let it be obſerved, that the ſpring months are thoſe that are juſt above the annulus, or circle of the ſigns in the ecliptic, and proceed from the left to the right hand, to the 10th of June; the ſummer months paſs thence from the right hand to the left to the 11th of September; then the autumn months proceed from the left to the right to the 10th of December; and the winter months return again from the right to the left; and this progreſs and regreſs appears plainly from the increaſe or decreaſe of the figures, pointing out the reſpective days of the months. And ſince, in the preceding ſcheme, not only the days of the ſun's ingreſs into, and egreſs out of, the ſeveral ſigns in the Zodiac are given, but alſo, whether his declination is increaſing or decreaſing, in every Quadrant of the circle, nothing more remains to ſhew which is the right ſign, whereto the Quadrant refers, for the ſun's place in the ecliptic.

PROBLEM III.

To find the right aſcenſion of the ſun.

Prob. 3. Right aſcenſion. Let the day be, as before, the 27th of April; lay the thread on this day, in the uppermoſt circular arcs of months, and it will cut the line of ſines on the limb of the Quadrant, at 45 degrees; theſe degrees, while the ſun is departing from the equinox towards the tropics, muſt be counted according to the graduations on the limb, from the left edge to the right; but, when the ſun is returning from the tropics, then it is to be counted from the right edge to the left; which alterations are diſcoverable by the progreſs or regreſs of the days of the month; after which, the right aſcenſion, thus found, muſt be eſtimated according to the ſeaſons of the year, *viz.*

Collins, 16, 17.

From the 11th of June to the 13th of September, there muſt be added — —	90°
From September 13, to December 11, more —	90
From December 11, to March 10, more —	90
	270
	90
Which, with from March 11, to June 11, being 90°, make in all — — —	360

The

The right aſcenſion thus found and eſtimated, will agree and correſpond with the ſun's right aſcenſion on the equinoctial line. Thus in the preſent caſe of the ſun's right aſcenſion on the 27th of April, the ſun being in the firſt Quadrant, *viz.* 17° 7′ of Taurus, the thread cuts the line of ſines on the limb at 45°, as above, and nothing is to be added.

If you would convert the degrees into time, the ſame is done in this caſe, and all others, in this manner; divide the degrees by fifteen for hours; multiply the remainder by four for minutes of time; and divide the minutes of a degree (if any) by fifteen, for minutes of time to be added to the former. However, this trouble may be ſaved as far as ninety degrees, or ſix hours in time, by having recourſe to the line of hours underneath the line of ſines in the limb: for, as the number of degrees of the ſun's right aſcenſion is ſhewn in this line of ſines, ſo in the line of hours underneath, is ſhewn the hours of the ſame aſcenſion, taking the hours from the left edge of the limb to the right, when the ſun is departing from the equator to the tropics; but, from the right towards the left, when he is returning from the tropics to the equator.

In the preſent caſe, the ſtring laid over 45° on the limb, cuts between three and nine, or three hours from noon. But note, that the ſun's right aſcenſion, his place and declination, as found in the three foregoing problems, agree only in the ſecond year after leap year; and, in order to find the true numbers for any other year, the declination muſt be corrected by the rectifying table, near the centre of the Quadrant, and the thread laid over the corrected declination, will ſhew likewiſe the corrected right aſcenſion and place of the ſun for that day.

PROBLEM IV.

To find the oblique aſcenſion.

To find the oblique aſcenſion ſeveral rules are given, amongſt which there are ſome that can be reſolved by Inſpection on the Quadrant; others only by the reſolution of aſtronomical problems; but as theſe are moſt properly connected with thoſe aſtronomical problems, the ſame are reſerved till we come to treat thereon; where, from the comparing of the practice (by the rules for ſolving theſe problems) with the ſolution of them (by inſpecting the Quadrant) it will moſt plainly appear, how much eaſier the ſame are reſolved by Inſpection, than by thoſe rules laid down for that purpoſe. Prob. 4. Oblique aſcenſion.

PROBLEM V.

To find, by Inspection, the time of the sun's rising and setting, and, with these, his declination also.

Prob. 5. Sun's rising, setting, &c.

Let the day be, as before, the 27th of April. Lay the thread over that day on the two lowest circular arcs of months, and it cuts the lowest arc in the limb, from the mark o, which stands (as in the margin) between VIII and IIII, counted towards the right hand, at 17° 7′, the north declination.

VIII,o,IIII

And it also cuts the strait line of versed sines at 112° 30′, and the hours underneath the versed sines, at half an hour after four (in the morning) for the sun's rising, and half an hour before eight (at night) for the sun's setting; for, if the said 112° 30′ are turned into time, the result will be the same as mark'd out in the line of hours.

	H.	M.
Thus, because 15 degrees are equal to an hour, divide therefore 112 by 15, and the quotient is 7 hours, with a remainder of 7 degrees. — —	07	00
And, because one degree is equal to 4 minutes in time, therefore multiply 7 by 4, and it gives — —	00	28
And, because 15 minutes of a degree is equal to one minute in time, therefore divide the remaining 30 minutes by 15, and it quotes — —	00	02
And the whole is — — — —	07	30

Or, half an hour before eight at night, and consequently half an hour after four in the morning.

PROBLEM VI.

To find the sun's ascensional difference.

Prob. 6. Ascensional difference.

As this imports no more than the time the sun rises before, or sets after six, you must therefore find the time of the sun's rising by the foregoing problem. Then observe how much the string laid over the time in the strait line of versed sines, is before or after six in the hour line, and so much does the sun rise before, or set after six, or, in other words, so much is his ascensional difference.

Ex-

EXAMPLE.

The ſtring laid over April the 9th, on the lower circular arcs of months, cuts the ſtrait line of verſed ſines and hours underneath, between five and ſeven, which for the ſun's riſing is one hour, or ſixty minutes before ſix; and, for his ſetting, the like time after ſix; and that is his aſcenſional difference for this day. Example.

So again, on February the ſixth, the ſtring laid in the lower circular arcs of months, cuts the ſtrait line of hours between ſeven and five. Wherefore, the time of the ſun's riſing is at ſeven, or one hour after ſix, and his ſetting is at five, or one hour before ſix; and in each caſe the aſcenſional difference is one hour.

PROBLEM VII.

To find the length of the day or night.

By the fifth problem find the time of the ſun's riſing and ſetting; then the time of his ſetting being doubled, gives the length of the day; and the time of his riſing doubled, gives the length of the night. Prob 7. Length of the day and night.

ALTI-

ALTIMETRIA;

OR,

The Uses of the Lines *on the* QUADRANT, *in the measuring or taking of Heights, according to the Rules of Trigonometry.*

Uses of the lines.

IN measuring or taking of heights, the line of equal parts on the left edge of the Quadrant is absolutely necessary.

This line is divided into ten equal parts, or principal divisions, called Primes, and these are sub-divided into ten other equal parts, called tenths, and each of these into halves.

The figures 1, 2, 3, to 10, by which the primes are distinguished, are arbitrary, and may each of them (as occasion requires) be made to represent so many units, tens, hundreds, or thousands; or they may represent so many tenth, hundredth, thousandth parts of an unit. Now, when the prime represents ten units, then each tenth or stroke between that and the next prime, will be an unit, and the stroke between each of these will be one half of an unit.

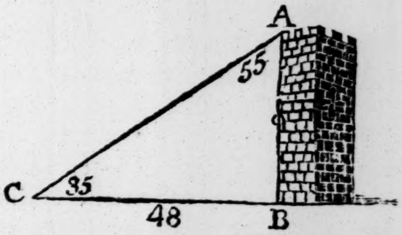
Again, if the prime represents an hundred, then the figures 2, 3, 4, *&c.* will denote 200, 300, 400, *&c.* and, according to this estimation, each tenth or stroke between them will be ten units; and, as there is but one stroke between each of these tenths, the same will be five units, or an half of the stroke denoting ten.

To apply this, suppose 125 was to be measured or taken on the line of equal parts, it would run beyond the limit of the Quadrant, which does not reach further than ten. Wherefore, I consider 1, in the prime or principal divisions as 100, and 2 divisions in the lesser or intermediate divisions, as tens; and the stroke in the middle as five units.

Let us now proceed to apply this notation to Altimetria, the present subject.

Suppose

Suppose it was required to take the height of an accessible tower, steeple, tree, or other object.



1. Draw at pleasure (if you would do this geometrically) a base line, as CB; and suppose B to be the foot of the tower.

2. Measure the distance between the foot of the tower at B, and the station of the observer at C, and lay off that distance (suppose 48 yards) on the base line from B, by the help of the scale of equal parts.

3. Erect a perpendicular at B, of an indefinite length.

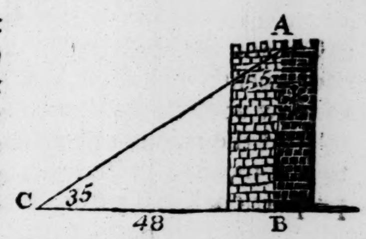
4. With the Quadrant, at the station point C, look at the top of the tower through the two sights, and observe what degree is cut by the line and plumbet, on the limb of the Quadrant; suppose this to be 35°, then, consequently, the angle at A will be 55°.

5. If at C you make an angle of 35°, with a Protractor, or other instrument, and draw the leg of that angle CA, then the point A, where this line cuts the line AB, will represent the top of the tower; and the line AB, measured on the same scale of equal parts with the line CB, will give the height of the tower, which, in this case, will be 33 yards and 2 feet.

6. But, if we are to find the height of the tower by the lines of the Quadrant, the case will stand thus.

PROBLEM I.

Prob. 1.



In the triangle ABC, right angled at B, are given, (beside the right angle) the base BC 48 yards; the angle at C, 35°, and, consequently, the angle at A 55°; thence to find AB, the height of the tower. Now, according to the first case of right angled plane triangles, as the sine of the angle at A, 55 degrees, is to 48 yards, the measure of BC, so is the sine of 35°, the angle at C, to AB the fourth proportional, which will appear to be 33 yards and two feet; to which adding the height of the Quadrant from the ground, suppose five feet, you have the height of the tower, 35 yards and one foot.

1st Hodgson, 104, 105.

Practice on the QUADRANT.

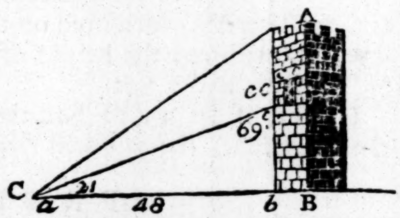
Take 48 on the line of equal parts in the compaſſes, with that extent enter one foot at the ſine of 55 degrees, and bring the thread to the other foot, keeping it in that poſition. Then ſet one foot of the compaſſes (diſcharged of their firſt extent) to 35° on the ſines, and with the other take the neareſt diſtance to the thread. This extent, applied to the centre of the equal parts, will reach to 33 yards two feet; to which, adding the height of the Quadrant, five feet, gives 35 yards one foot, as before.

Collins 36. *Euclid* 16. Another way to take the altitude of the tower AB, at one ſtation. Go ſo far back from the foot of the tower, in a ſtrait line, as that, looking through the two ſights of the Quadrant, the thread may cut the limb at 45°. Then meaſure the diſtance from the foot of the tower, to the ſtation where that angle was taken; and this diſtance will be equal to the height of the tower, above the eye.

PROBLEM II.

To meaſure part of an altitude, as ſuppoſe from a window in a tower, to the top of the tower.

Problem 2d Suppoſe the height of the tower, found as in the firſt example, to be 35 yards and one foot, and ſuppoſe there is a window at CC, then to gain the height of the window, you have given, in the inner or leſſer triangle *a b c*, 48 yards equal to



a b, the meaſured diſtance from the foot of the tower, to the ſtation at C; and the angle at C, taken by the Quadrant, equal to 21°, and conſequently the angle at CC, 69°, thence to find the height of the window at CC.

And the rule is as before. As the ſine of the angle at CC 69°, to the meaſured diſtance 48 yards; ſo is the ſine of 21°, to the 4th proportional, 18 yards, one foot, nearly, on the equal parts.

Practice on the QUADRANT.

Becauſe the firſt term, 69°, is greater in meaſure than the ſecond, *viz.* 48 equal parts. Take 48 on the line of equal parts in the compaſſes; enter one foot of them at 69° on the ſines, and bring the thread to the other foot. Then ſet one foot of the compaſſes (diſcharged of their firſt extent) to the ſine of 21°, and there take the neareſt diſtance to the thread; apply this diſtance to the centre of the

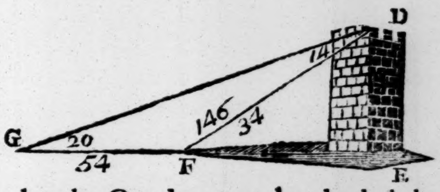
equal

equal parts, and it will reach to 18 yards, one foot, nearly; to which adding the height of the eye, above the ground, *viz.* one yard two feet, it will make twenty yards, the height of the window, which deducted from the height of the tower, 35 yards 1 foot, leaves 15 yards 1 foot for the distance between the tower's top, and the window.

PROBLEM III.

Let it be required to take the height of an inaccessible tower.

Problem 3d.



In order to solve this Problem, suppose a base line G E, drawn at pleasure, and a perpendicular erected at E. Then any where, in the line G E, to which you can have access, suppose at F, take your first station, and there, by the Quadrant, take the height of the tower, which, suppose, by the line and plumbet, to cut at 34° on the limb, in the small figures there, which best describe the inward angle, as in this case. Then at this station, F, draw the line F D, forming the above angle of 34°, and this will intersect the perpendicular D E, at D the top of the tower. Set a staff, or some other mark, at this station, *viz.* at F, and then carry the Quadrant back in a strait line, to some other accessible place, in the base line, suppose to G, and there look again through the two sights of the Quadrant, at the point D, the top of the tower, and note the degrees which the thread cuts at the limb of the Quadrant, suppose 20 degrees. Now the angle at F, being found (as above) to be 34°, its supplement D F G is 146 degrees, and the angle at G, being found to be 20°, it follows that the angle F D G is 14°.

Collins, 42, 54. *Deschales's Euclid, 1st p.* 16, 20, 55.

And now we have got all the angles, in the triangle D F G, as in the margin; and these being thus found, it may be thought we have sufficient to pronounce the lengths of the three sides; but it is intimated, in the beginning of this discourse, that the knowledge of the three angles of any triangle, does not give the measure, but only the proportion of the sides. Therefore we must find out the measure of one of the sides, and this we may obtain readily, if we take or find the measure of the side, containing the distance between the two stations G and F. Measure, therefore, this distance, and let it be supposed that it comes out to be 54 feet. Then say, according to the second case of oblique angled plane triangles, As the sine of the angle at D in the triangle D F G = 14°, is to 54 feet, so is the sine

DFG.	146
DGF.	20
FDG.	14
	180

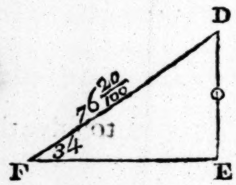
1st *Hodgson,* 120.

of

of the angle at G, 20°, to the fourth proportional, which will appear to be 76 feet $\frac{20}{100}$ the side D F.

And now having got F D, the side of the triangle D E F, the way is cleared to find the side D E, the height of the tower; for in this triangle D E F right-angled at E, are given the hypotheneuse $= 76\frac{20}{100}$, and the angle at F, 34°; thence to find the leg D E, the height of the tower, it

1st, *Hodgson*, 110. will be (according to the third case of right-angled plane triangles) As the radius to the sine of the angle D F E, 34° so is $76\frac{20}{100}$, to the fourth proportional, which will be $42\frac{40}{100}$ feet, the height of the tower, seen from the quadrant, at the height of five feet; which (being added to the said $42\frac{40}{100}$) make together, the height of the tower D E, 47 feet $\frac{40}{100}$.



The Practice on the QUADRANT, *in the first proportion used in the preceding case*, viz.

I. As the sine of the angle at D in the triangle G D F, 14°, is to 54; so is the sine of the angle at G, 20°, to $76\frac{20}{100}$.

Take 14° on the line of sines in the compasses, and enter one foot of them, with that extent, at 54, on the line of equal parts, and bring the thread to the other foot: Then take 20°, on the line of sines, in the compasses, and enter them with that extent, between the thread and the equal parts, and they will rest at $76\frac{20}{100}$.

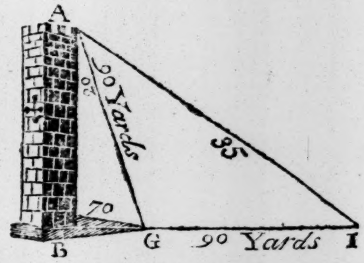
The Practice on the QUADRANT, *in the second proportion*, viz.

II. As radius, to the sine of the angle D F E, 34°; so is $76\frac{20}{100}$, to the fourth proportional, may be as follows; because the last term, in this proportion, will be taken on the line of equal parts: Therefore, take the sine of 34° in the compasses, and with that extent enter one foot at radius (or ten) in the equal parts, and bring the thread to the other foot. Then apply one foot of the compasses (discharged of the former extent) to $76\frac{20}{100}$, on the line of equal parts; and thence, with the other, take the nearest distance to the thread: This extent (applied to the centre of the equal parts) will reach to $42\frac{40}{100}$ the height of the tower, above the height of the Quadrant.

PRO-

PROBLEM IV. Prob. 4.

Being another method of performing Problem the third; to find the altitude of an inacceſſible tower, at two ſtations.



Suppoſe the firſt ſtation any where, as at G; and the angle of the altitude of the tower, there obſerved through the two ſights of the Quadrant, to be 70 degrees; if you remove ſo far back, ſuppoſe to I, as that the object may appear but half ſo high, *viz.* at 35°; then the diſtance between theſe two ſtations, G and I, is equal to the length of the hypothenuſe AG; ſuppoſe, therefore, the diſtance between G and I, being meaſured, is found to be 90 yards; then is the hypothenuſe AG alſo 90 yards; and then, in the triangle ABG right-angled at B, the angle at G being found to be 70°, conſequently the angle at A is 20°, and the ſide or hypothenuſe 90 yards; thence to find AB, the height of the tower, the rule is (according to the third caſe of right-angled plane triangles, in 1ſt *Hodgſon*, page 110.) As radius to the hypothenuſe, 90 yards, equal to the meaſured diſtance, ſo is the ſine of 70°, the angle AGB, to the height of the tower AB, 84 yards.

Collins 40, 41.

The Practice on the QUADRANT.

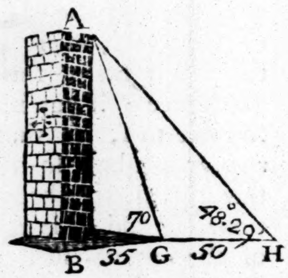
Becauſe the laſt proportional is to be taken on the line of equal parts; therefore, take 70° on the line of ſines in the compaſſes, and (with that extent) enter one foot at radius (or ten) on the line of equal parts, and bring the thread to the other foot; then enter one of the legs of the compaſſes (diſcharged of their firſt extent) at 90 on the line of equal parts, and take the neareſt diſtance to the thread; this extent, applied to the centre of the line of equal parts, will reach to 84 yards, the height of the tower.

Prob. 5.

PROBLEM V.

Being a third way of performing Problem the third; to find the height of an inacceſſible tower, by a more general way, with two ſtations taken at random.

Collins 154. 42. *Oughtred's Proportions*, 246.



Suppoſe the firſt ſtation to be at G, and that looking through the ſights of the Quadrant, the thread cut the limb at 70°: Again, ſuppoſe at the ſecond ſtation at H, the angle was 48° 29′. Alſo, ſuppoſe the diſtance meaſured between theſe ſtations G and H, to be 50 yards, then the proportion, to attain the altitude of the tower, will be, As the difference of the cotangents of the angles, found at the two ſtations, is to the diſtance between the two ſtations, 50 yards; ſo is radius to the altit ude of the tower, 96 yards.

The complement of	48° 29′	is	— — —	41° 31′
The complement of	70	is	— — —	20 00.

The Practice on the QUADRANT.

In this caſe, the queſtion will be anſwered more eaſily, by changing the middle terms; then, becauſe the laſt term is on the line of equal parts, take in the compaſſes the diſtance on the line of tangents, between 41° 31′ and 20°, and enter one foot of the compaſſes with this extent at the radius or ten on the equal parts, and bring the thread to the other foot; then (diſcharging the compaſſes of their firſt extent) take 50, or 5, on the line of equal parts; and entring this extent between the ſcale of equal parts and the thread, the compaſſes will reſt at 96 on the ſame line, the height of the tower required.

PRO-

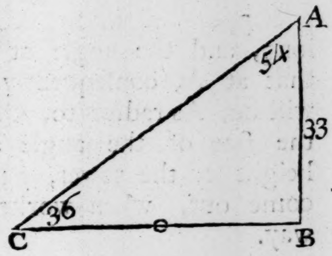
PROBLEM VI.

Prob. 6.

From a tower whose perpendicular height is given, to take the distance (between B *and* C) *the foot of the tower, and any object at a distance therefrom.*

EXAMPLE.

Suppose the given height of the tower AB to be 33 yards, the angle observed at A to be 54°, and consequently the angle at C to be 36°; thence to find the distance required: Then by the first case of right-angled triangles, As the tangent of the angle at C 36° to the radius; so is AB, 33 yards; to BC, 45 yards, and something more.



1st *Hodgson*, 1041.

The Practice on the QUADRANT *in the foregoing* Example.

Take 36° on the tangents, in the compasses; with that extent set one foot to radius (or 10) on the equal parts, and bring the thread to the other foot; then take 33, on the line of equal parts, from the centre, and (with that extent) enter the compasses, between the thread and the equal parts, and it will rest at 45 on the equal parts, nearly the distance BC required.

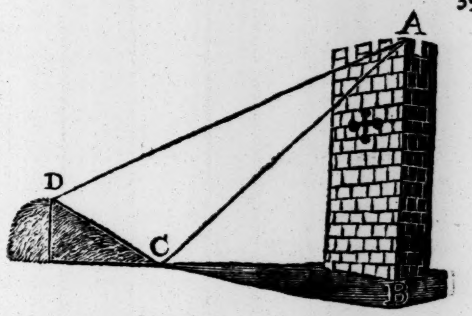
PROBLEM VII.

Prob. 7.

It is required to find the measure of an inaccessible height AB, *placed so that one can neither go near it, in an horizontal plane, nor recede from it.*

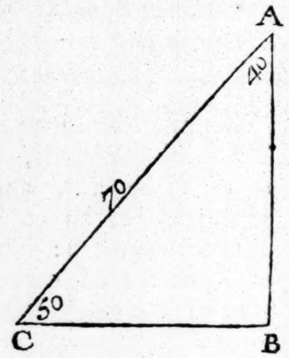
Dr. *Gregory's Geomet.* 39.

To resolve this case, let AB be the height of the tower; let there be chosen any situation as C, and another as D, where let some mark be erected; let the angles ACD and ADC, be taken by the Quadrant; and then the third angle CAD is known: Let the Side CD, the distance between the two stations be measured; and then the side AC will be found by the rules of oblique-angled plane triangles.



Again,

Again, in the triangle A B C, right-angled at B, having found by the Quadrant the angle ACB, the other angle CAB is known likewise.



But the side AC, in the triangle ADC, is already known; therefore the height required, A B, can be found by the third case of right-angled plane triangles.

Suppose, therefore, CA to be 70 poles long, and the angle at C to be 50°, and that at A, consequently, 40°; and then it will be, As radius to, CA, 70 poles; so is the sine of the angle at C 50°, to the height of the tower, 53 poles; as it will come out, when worked in the common way.

NAVIGATION.

Several PROBLEMS and CASES of plane ſailing, relating to a ſingle courſe, practically ſolved by the QUADRANT.

1ſt, *Hodgſon's* Theory of Navigation, 145.

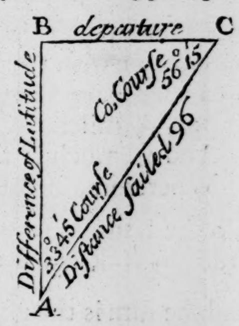
CASE I.

The courſe, one latitude, and the diſtance ſailed, being given; to find the other latitude, and the departure from the Meridian. Caſe 1.

EXAMPLE.

A Ship at ſea, in the latitude 46° 30′ north, ſails 96 miles, upon the third rhumb of the compaſs, or north-eaſt by north = 33° 45′; the latitude the ſhip is in, and how much ſhe has departed from her former meridian, is required? Let A B repreſent the meridian, and (for method's ſake) the upper-end towards B the north, and A the ſouth part thereof; then will that part next the right hand, be the eaſt, and that towards the left hand the weſt; which order will be obſerved in the ſucceeding caſes; alſo, let A be the place the ſhip departed from.

The compaſs with its rhumbs are given in 1ſt *Hodgſon*, 141, 142.



Then, in the triangle ABC, are given the hypotheneuſe, A C, 96 miles; and the angle BAC = 33° 45′; to find the departure or meridional diſtance. And it will be (according to Caſe the third of right-angled plane trigonometry) As radius to the diſtance, 96 miles; ſo is the ſine of the courſe, 33° 45′; to the departure BC, 53° $\frac{84}{100}$ miles; as it will appear. 1ſt *Hodgſon*, 109, 110. 145.

The

The Practice on the QUADRANT, *in the foregoing Case.*

Because the laſt proportional is to be taken on the line of equal parts, therefore take 33° 45′ from the line of ſines, in the compaſſes, and apply one foot (with that extent) to radius (or ten) on the equal parts, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 96, on the ſame line of equal parts, and with the other take the neareſt diſtance to the thread. This extent (applied to the centre of the line of equal parts) will reach to 53 $\frac{34}{100}$ miles, and ſo much has been the ſhip's de-
1ſt *Hodgſon*, 147. parture to the eaſtward. If the courſe had been weſterly, then the former number would have ſhewn how much the ſhip was got to the weſtward; and the ſame meridional diſtance will obtain in ſailing (from any point on the globe) 96 miles upon the third rhumb.

Secondly, to find the alteration or difference of latitude, from the
Idem, 109. 147. ſame data as before, As radius, to the diſtance ſailed, 96 miles; ſo is the co-ſine of the courſe = 56° 15′, to the difference of the latitude; which will appear to be 79 $\frac{82}{100}$ miles.

The Practice on the QUADRANT.

Becauſe the laſt proportional is to be taken on the line of equal parts, take 56° 15′ from the ſines, in the compaſſes, and (with that extent) enter one foot at radius (or 10) on the line of equal parts, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 96, on the equal parts, and with the other take the neareſt diſtance to the thread; then that extent (applied to the centre of the equal parts) will reach to 79 $\frac{82}{100}$ miles.

And ſo much is the ſhip got to the northward of her laſt place.

Wherefore, becauſe ſhe is 79 $\frac{82}{100}$ miles further from the equator,

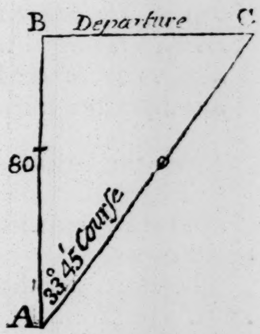
To the latitude ſailed from, *viz.* — — —	46° : 30′ N.	
Add 79 $\frac{82}{100}$, equal to — — — — —	1 : 19 $\frac{82}{100}$,	
and the ſum is the north latitude which the ſhip is in,	47 49 $\frac{82}{100}$.	

If the courſe had been ſoutherly, the ſhip would then have been gotten 79 $\frac{82}{100}$ miles to the ſouthward of her former place, and therefore, in this caſe, to find the latitude the ſhip is in; from the latitude ſailed from 46° 30′ north, take the difference of the latitude made

made 1° 19′ $\frac{82}{100}$ south, and there remains the present latitude 45° 10′ $\frac{18}{100}$ north.

CASE II.

A ship at sea, being in the latitude of 46° 30′ north, after having sailed some time upon the third rhumb, or north-east by north = 33° 45′, is found, by observation, to be in the latitude 47° 50′ north. The distance sailed, and departure from the meridian, are sought? Case 2.



In this triangle ABC right-angled at B, are given the angle of the course at A 33° 45′; and because the latitudes are both north, and the ship's course northerly, if from the latitude found by observation 47° 50′, you subtract the latitude sailed from, 46°, 30′, there remains the difference of latitude made, *viz.* 1° 20′, or 80 miles, the length of the leg A B. These two parts of the above right angled triangle being known, the rule laid down by Mr. Hodgson, page 148, 108, for finding the departure, is, As radius, to the difference of latitude, *viz.* 80 miles, so is the tangent of the course, 33° 45′, to the departure, which will appear to be 53 $\frac{45}{100}$ miles.

Practice on the QUADRANT.

Because the last proportional will be upon the equal parts, take 33° 45′ from the scale of the tangents in the compasses, and with this extent enter one foot at radius (or 10) on the equal parts, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at 80 on the line of equal parts, and with the other take the nearest distance to the thread. This extent (applied to the centre of the equal parts) will reach to 53 $\frac{45}{100}$, the departure BC, and so much is the ship got to the eastward.

Secondly, for the distance sailed, the rule given by Mr. Hodgson, Vol. I. page 108, 149, is according to the second case of right angled plane triangles; As radius, is to the difference of latitude, 80 miles, so is the secant of the angle at the course, 33° 45′, to the direct distance 96 $\frac{21}{100}$; set the string to 33° 45′ on the line of secants: then, from the centre of the Quadrant, take 80 (upon the equal parts) in the compasses, and entring them with that extent, between the scale of equal parts and the thread, they will rest at 96 $\frac{21}{100}$ miles, the distance sailed.

 This

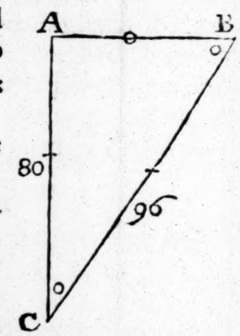
This Problem may be solved without the secants, and by this rule.

As the co-sine of the course 33 45 = 56° 15′
To the radius,
So is 80 miles
To $96\frac{21}{100}$; to be performed the common way.

CASE III.

Case 3. *A ship at sea, in the latitude of 46° 30′ north, sails upon some rhumb, between the north and east, 96 miles; and then is found, by observation, to be in the latitude of 47° 50′ north; the true course steered, and departure from the meridian are required.*

From the latitude found by observation — — 47° 50′ north,
Take the latitude sailed from, *viz.* — — — 46° 30′ north,
And there remains the difference of latitude sailed, 1° 20′ north,
or 80 miles.



Therefore, in the triangle ABC, right angled at A, are given BC the distance sailed, equal to 96 miles, and the difference of latitude AC = 80 miles; to find the course and departure.

And first to find the course, it will be by the rule laid down by Mr. Hodgson page 113, 114, 150, for the fifth case of right angled plane triangles; As the distance sailed, 96 miles, is to the radius, so is AC, the difference of latitude, 80, to the co-sine of the course, which will be 33° 33′; and, because the ship sailed between the north and the east, the course is north 33° 33′ east, or north east by north nearly.

The Practice on the QUADRANT.

Here, because the last proportional is to be found on the sines, therefore take 96, from the line of equal parts, in the compasses; enter one foot, with that extent, at the radius, on the sines, and bring the thread to the other; then take 80 from the line of equal parts in the compasses; with that extent, enter it between the line of sines and the string, and it will rest at 56° 27′, the complement of 33° 33′, the true course.

Then, to find the departure, it will be according to the rule in
1st *Hodg-* 1st Hodgson, 151, for the third case of right angled trigonometry,
son, 151. 109, As radius, to the distance sailed, 96 miles, so is the sine of the

the course, 33° 33′, to the departure, which will be 53 $\frac{6}{100}$, eastwardly.

The Practice on the QUADRANT.

Change the two middle terms; and, because the last proportional is to be taken on the equal parts, take from the sines 33° 33′, the angle at C, between the compasses; with this extent, enter one foot at the radius (or 10) on the equal parts, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at 96 on the line of equal parts, and, with the other, take the nearest distance to the thread; apply one foot of the compasses, with that extent, to the centre of the equal parts, the other will reach to 53 $\frac{6}{100}$ miles.

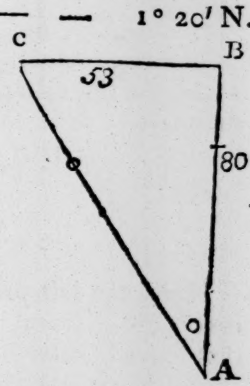
The latitudes of any two places being given, and their meridional distance, the following case is of use, to determine the true rhumb the ship is to sail upon, and how far.

CASE IV.

A ship at sea in the latitude 12° 10′ north, is bound to Barbadoes in the latitude 13° 30′ north, the meridional distance, by estimation, being 53 miles west; the direct course and distance from the ship to her port, are required. Case 4.

From the latitude of Barbadoes — — —	13° 30′ N.
Take the latitude the ship is in, — — — —	12° 10′ N.
And there remains — — — — — —	1° 20′ N.

equal to 80 miles, for the difference of the latitude AB, and the departure B C, 53 miles, being given, thence to find the course or angle B A C.



And the rule for this is, according to the fourth case of plane Trigonometry; As the difference of latitude, 80 miles, to the radius, so is the departure B C, 53 miles, to the tangent of the course 33° 31′, as it will appear. 1st Hodgson, 151, 152, 110.

The Practice on the QUADRANT.

Here, because the last proportional is to be taken on the tangents, take 80 from the line of equal parts, in the compasses; enter, with this extent, one foot at radius on the tangents, and bring the thread

 to

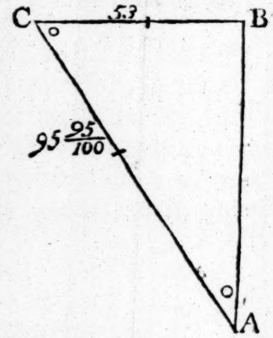
to the other; then take 53, from the line of equal parts, in the compasses (discharged of their first extent) and entring them between the scale of tangents and the string, they will rest at the tangent of 33° 31′, the ship's true course, which is north 33° 31′, west or north west by north, nearly: Then, to find the direct distance, it will be (according to the second case of plane trigonometry in 1st *Hodgson*,
1st *Hodg-son*, 152. 108.) As the radius, to the difference of latitude, 80 miles; so is the secant of the course, 33° 31′, to 95° $\frac{95}{100}$, the direct distance, as it will appear.

Practice on the QUADRANT.

Set the string to 33° 31′ on the secants; then take 80, the difference of the latitude, in the compasses; and with that extent (entring it between the thread and the line of equal parts) it will rest at 95 $\frac{95}{100}$. Or this may be performed without the secants, thus: As the sine of 56° 15′ is to radius, so is 80 to 95 $\frac{95}{100}$.

CASE V.

Case 5. *A ship at sea, in the latitude of 12° 10′ north, having sailed between the north and west 95 $\frac{95}{100}$ miles; and having made 53 miles of westing; the direct course steered, and the latitude the ship is in, are required?*

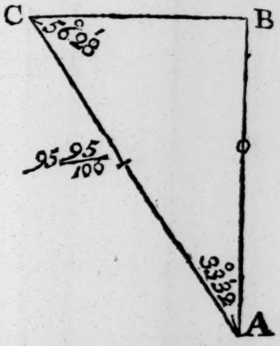


In this triangle ABC, the distance sailed AC, is 95 $\frac{95}{100}$ miles, and the departure BC 53 miles; then to find the true course, the rule is (according to the fifth case of right-angled trigonometry, 1st *Hodgson*, page 153,
1st *Hodg-son*, 153, 114. 114.) As the distance sailed, 95 $\frac{95}{100}$ miles, to the radius; so is the departure, 53 miles, to the sine of the true course 33° 32′, as it will appear.

The Practice on the QUADRANT.

Here, the last proportional being to be taken on the line of sines; take 95 $\frac{95}{100}$, from the line of equal parts, in the compasses; with that extent, enter one foot at radius, on the sines, and bring the thread to the other foot; then take 53 from the line of equal parts, in the compasses; enter with that extent, between the line of sines and the thread; and they will rest at 33° 32′, the sine of the true course, which is north-west by north, nearly.

And,



And, to find the difference of latitude, it will be (by cafe the third of right-angled triangles.) As radius, to the diftance failed, 95 $\frac{95}{100}$ miles; fo is the cofine of the courfe, 33 32 = 56 28; to the difference of latitude, 79 $\frac{98}{100}$ or 80 miles, nearly, as it will appear. 1ft *Hodgfon*, 154. 109.

The Practice on the QUADRANT.

Here, becaufe the laft proportional, to be taken, is, on the line of equal parts, take 56° 28′, the cofine of the angle BAC, in the compaffes; enter one foot of them, with that extent, at radius (or ten) on the equal parts, and bring the thread to the other foot; then enter one foot of the compaffes (difcharged of their firft extent) at 95 $\frac{95}{100}$ on the line of equal parts; thence take the neareft diftance to the thread; and applying that extent to the centre of the equal parts, it will reach to 79 $\frac{98}{100}$: Now, to find the latitude the fhip is in, becaufe fhe failed from a north latitude, northerly, add to the latitude, from which fhe failed, *viz.* 12° 10′ north, the difference of the latitude (79 $\frac{98}{100}$ or) 80 = 1° 20′ north; the fum will be the latitude the fhip is now in, 13° 30′. 1ft *Hodgfon*, 154.

Thus much as to fuch cafes as concern a fingle courfe: And, as compound courfes or traverfes, confift of feveral fingle courfes, the folutions of which depend upon one or other of the preceding cafes of right angled trigonometry, which may be refolved, practically, by the Quadrant; it is, therefore, of no ufe, here, to go again into the fame matter, and do again what is already done.

Various cafes are put and anfwered, in practical navigation, by oblique-angled plane triangles; all of which may be anfwered very readily, by the Quadrant; and as the folutions of all the fix cafes, in oblique-angled plane triangles, are given in the firft volume of Mr. *Hodgfon*'s fyftem (beginning at page 119.) Alfo as the cafes in navigation, depending upon thefe folutions, (which may be called coafting cafes) are feldom wanted; I choofe to break off here, and proceed to the folution of aftronomical problems, by fpherical triangles; in which, all the cafes of any difficulty, that may happen, in the practifing by the Quadrant, will be cleared. 1ft *Hodgfon*, 204, &c.

The

The Application of Spherical Trigonometry to the practical Solution of the chief PROBLEMS of ASTRONOMY, by the QUADRANT.

And first, to those Cases that relate to the Sun, which for the better, and more ready finding them, are digested into an alphabetical Order, as follows:

Altitude of the Sun.

PROBLEM I.

Prob 1. *Given the* Sun's *azimuth at the hour of six, and his declination, to find his altitude.*

EXAMPLE.

2d *Hodgson*, 298, 110. THE sun having 19° 39′ north declination, and his azimuth at 6 being 77° 29′, his altitude is required; and the rule, to find it, is (according to the second example of the fourteenth case of right-angled spherical triangles,) as the sine of the azimuth 77° 29′ to the radius, so is the cosine of the declination 19° 39′; to the cosine of the altitude 15° 16′.

The Practice on the QUADRANT.

Take 77° 29′ from the scale of sines in the compasses, with that extent, enter one foot at the radius, and bring the thread to the other; then take 70° 21′, the complement of 19° 39′, in the compasses, from the centre of the sines, and entring, with that extent, between the thread and the line of sines, they will rest at 74° 44′, the complement of the altitude 15° 16′, which was required.

PROBLEM II.

The latitude of the place, and the Sun's declination being given; to find his height when on the prime vertical. Prob. 2.

EXAMPLE.

Given the latitude 51° 32′, and the sun's declination 19° 39′, to find his height, when due east or west; the rule is (according to case the tenth of right-angled spherical triangles,) As the sine of the latitude 51° 32′ is to radius, so is the sine of the sun's declination, 19° 39′, to the sine of his height in the prime vertical 25° 26′. 2d *Hodgson*, 303. 97.

The Practice on the QUADRANT.

Take 51° 32′ from the sines in the compasses; and, with that extent, enter one foot at the radius on the sines, and bring the thread to the other; then take 19° 39′ in the compasses, from the centre of the sines, and entring with that extent between the thread, and the line of sines, they will rest at 25° 26′ his altitude on the prime vertical; which altitude will be the same whether the sun appears due east or west.

PROBLEM III.

The sun, being upon the prime vertical, there are given, the declination, and the time of the day, to find the altitude. Prob. 3.

EXAMPLE.

The sun, having 19° 39′ of north declination, was observed to be upon the prime vertical, or due east or west, at 54 minutes after four in the afternoon; his altitude is required; the rule is (according to the first case of right angled spherical triangles) As radius to the co-sine of the sun's declination, 19° 39′, so is the sine of the hour from noon, (converted into degrees) = 73° 31′, to the co-sine of the altitude 25° 26′, as it will appear. 2d *Hodgson*, 308, 66.

The Practice on the QUADRANT.

Take 17° 21′, the complement of the declination, from the sines, in the compasses, and, with that extent, enter one foot at the radius on the sines, and bring the thread to the other; then set one foot of the compasses (discharged of the first extent) to 73° 31′ on the sines, and,

and, with the other, take the nearest distance to the thread; this extent, applied to the centre of the sines, will reach to 64° 34′, the complement of the altitude 25° 26′.

PROBLEM IV.

Prob. 4. *The sun being in any point of the ecliptic, his declination being given, together with the hour of the day, and latitude of the place; to find the sun's altitude.*

Example.

2d *Hodgson*, 323, 197. In the latitude of 51° 52′ north, the sun having 19° 39′ north declination, what will his altitude be, at half an hour past eight in the morning, or at half an hour past three in the afternoon, at each of which times, he is equally distant from the meridian.

Before we can solve this Problem, the sun's azimuth must be found; to do which, two proportions must be taken: the rule for the first of which is (according to the tenth case of oblique spherical triangles) As radius, to the co-sine of the hour from noon, 52° 30′, so is the co-tangent of the declination, 19° 39′, to the tangent of a fourth arc, which will appear to be 59° 36′.

The Practice on the Quadrant, *in this proportion.*

Because the tangent of 59° 36′ exceeds the bounds of the Quadrant, the proportion may be taken thus, As the sine of 37′ 30′, the complement of 52° 30′, the hour from noon, to the radius, on the tangents, so is 19° 39′, the tangent of the declination, to the tangent of 30° 24′, the complement of the fourth proportional = 59° 36′. Take 37° 30′, from the sines, in the compasses; with that extent, set one foot at the radius on the tangents, and bring the thread to the other; then take 19° 39′ from the line of tangents, and entring with that extent, between the thread and the line of tangents, the compasses will rest at 30° 24′, the complement of 59° 36′.

If from the fourth arc 59° 36′, thus found, be taken the complement of the latitude 38° 28′, there will remain a fifth arc, *viz.* 21° 08′.

2d *Hodgson*, 324, 190. 191. And then the rule for the next process is (according to the ninth case of oblique spherical triangles) As the sine of the fifth arc, 21° 08′, to the sine of the fourth arc, 59° 36′, so is the tangent of the hour from noon, 52° 30′, to the tangent of the azimuth from the meridian, 72° 13′, as it will appear.

5 *The*

The Practice on the QUADRANT.

Here, becaufe the tangents exceed the limits of the Quadrant, change the places of the firft and fecond terms; and this will infer a change of the two latter terms, into their co tangents; and then (changing the two middle terms) the analogy will be; As the fine of 59° 36′, to the co-tangent of 52° 30′, that is, the tangent of 37° 30′, fo is the fine of 21° 08′, to the tangent of 17° 47′, which is the co-tangent of 72° 13′, the azimuth required. *Leybourn's* Panorganon, 133. *Collins*, 73.

Therefore, in practice, take 37° 30′ from the tangents in the compaffes, and (with that extent) enter one foot at 59° 36′ on the fines, and bring the thread to the other; then enter one foot of the compaffes (difcharged of the firft extent) at 21° 08′ on the fines, and, with the other, take the neareft diftance to the thread; this extent, applied to the centre of the tangents, will reach to 17° 47′, the complement of 72° 13′, the azimuth required: which, in the prefent cafe, becaufe the latitude is north, and the time near noon, the azimuth muft be counted from the fouth point of the horizon; wherefore (if the time given was in the forenoon) the azimuth is fouth 72° 13′ eaft, or eaft fouth eaft $\frac{1}{4}$ eaft, nearly. But (if the time given be in the afternoon) the azimuth is fouth 72° 13′ weft, or weft fouth weft $\frac{1}{4}$ weft, nearly.

Having now found the azimuth of the fun, his altitude may be found by the tenth cafe of oblique fpherical triangles; As the co-fine of the fourth arc, 59° 36′, to the co-fine of the fifth arc, 21° 08′, fo is the fine of the declination, 19° 39′, to the fine of the altitude, 38° 19′. 2d *Hodgfon*, 325, 196.

The Practice on the QUADRANT.

Take 30° 24′, the complement of the fourth arc, from the fines in the compaffes, and enter one foot, with that extent, at 68° 52′, the complement of the fifth arc, on the line of fines, and bring the thread to the other; then take 19° 39′, in the compaffes, from the centre of the fines, and entring, with that extent, between the thread and the fcale of fines, they will reft at 38° 19′, the fun's altitude; which is the fame, both forenoon and afternoon.

The like problem may more briefly be performed by the verfed fines on the Quadrant. Thus, in the triangle propofed are given two fides, and the angle P included, to find the third fide; the given fides are the co-latitude = 38° 28′, the co-declination 70° 20′, and the given angle is the hour from noon, 52° 30′.

In this case take the sum and difference of the two sides, *viz.*

	38° 28′	70° 20′	
	70 20	38 28	
Their sum	108 48	31 52	Their difference.

Now, take the distance between 108° 48′, and 31° 52′, on the versed sines in the compasses, and enter one foot of the compasses, with this extent, at 180° on the same sines, and bring the thread to the other; then enter one foot of the compasses (discharged of its first extent) at 52° 30′, being the hour of the day, on the same sines, and with the other take the nearest distance to the thread; this extent, applied to the centre of the versed sines, will reach to 38° 19′, the altitude required.

PROBLEM V.

Prob. 5. *The latitude, declination, and azimuth given, to find the altitude.*

EXAMPLE.

2d *Hodgson*, 347. 161. In the latitude of 51° 32′ north, the sun having 19° 39′ north declination, and his azimuth being south 72° 13′ west, his altitude is required. Then, according to the third case of oblique spherical triangles,

I. As radius, 90° 00′, to the co-sine of the azimuth, 17° 47′, so is the co-tangent of the latitude, 38° 28′, to the tangent of a fourth arc, 13° 38′.

II. Again: As the sine of the latitude 51° 32′, to the sine of the declination, 19° 39′, so is the co-sine of the fourth arc (13° 38′=) 76° 22′, to the co-sine of a fifth arc (65° 20′=) 24° 40′.

The Practice on the QUADRANT, *in the first Proportion.*

Take 17° 47′ from the sines, in the compasses; with that extent, enter one foot at the radius, on the tangents, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at the tangent of 38° 28′, the complement of the latitude, and with the other take the nearest distance to the thread; this extent, applied to the centre of the tangents, will reach to 13° 38′, the fourth arc.

OPE-

OPERATION *in the second Proportion.*

Take 19° 39′, from the sines, in the compasses; with that extent, enter one foot at 51° 32′ on the sines, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at 76° 22′, the complement of 13° 38′, on the sines, and, with the other, take the nearest distance to the thread; this extent, applied to the centre of the sines, will reach to 24° 40′, the complement of 65° 20′. Now, if from this 65° 20′, there be taken the fourth arc, 13° 38′, the remainder will be equal to 51° 42′, and this, taken from 90°, or a Quadrant, will give 38° 18′ for the altitude.

Or, the fourth arc, 13° 38′, added to the fifth arc, 24° 40′, will give 38° 18′, for the altitude required.

PROBLEM VI.

Let it be required to find the altitude of the sun, when he will appear on the west north west, or east north east azimuth circle, at Barbadoes, in the latitude of 13° 30′ north, at the time of the summer solstice, or when the sun has 23° 29′ of north declination. Prob. 6.

In this case there are given, the complement of the latitude (13° 30′ =) 76° 30′, the complement of the azimuth (67° 30′ =) 22° 30′, and the sun's declination, 23° 29′, north; thence to find the altitude. 2d *Hodgson*, 348.

This requires two operations; the rule for the first being, As radius, to the co-sine of the azimuth, 22° 30′, so is the co-tangent of the latitude, 76° 30′, to the tangent of a fourth arc, 57° 54′.

Here, since the tangent of the fourth term, and co-tangent of the third term, exceed the bounds of the Quadrant, change the places of the first and second terms, and this will infer a change of the co-tangent in the third place, into a tangent; and of the tangent in the fourth place, into a co-tangent; and then the proportion will stand thus, As the sine of 22° 30′, to the radius, so is the tangent of the latitude, 13° 30′, to the co-tangent of a fourth arc, (57° 54′ =) 32° 06′. *Collins*, 73.

The Practice on the QUADRANT *in this Proportion.*

Take 22° 30′ from the sines, in the compasses; with that extent, enter one foot at the radius, on the tangents (the last angle being to be taken on the tangents) and bring the thread to the other; then take 13° 30′ on the tangents, in the compasses, and entring between

the ſcale of tangents and the thread, they will reſt at 32° 06′, the co-tangent of 57° 54′. Then again, the ſecond proportion will be,
2d *Hodgſon*, 349, 350. As the ſine of the latitude, 13° 30′, to the ſine of the declination, 23° 29′, ſo is the co-ſine of the fourth arc, (57° 54′=) 32° 06′, to the co-ſine of a fifth arc, (24° 53′=) 65° 07′.

The Practice on the QUADRANT.

Take 13° 30′, from the ſines in the compaſſes; with that extent, enter one foot at 23° 29′ on the ſines, and bring the thread to the other; then take 32° 06′, in the compaſſes, from the centre, and entering, with that extent, between the thread and the ſcale of ſines, they will reſt at 65° 07′, the complement of 24° 53′, the fifth arc. If to the fourth arc, 57° 54′, be added the fifth arc, 24° 53′, the ſum is 82° 47′, which, taken from 90°, will leave 7° 13′, for the leaſt altitude; but, if from the fourth arc, 57° 54′, be taken the fifth arc, 24° 53′, the remainder is 33° 01′, which, taken from 90°, will leave 56° 59′, for the greater altitude; and theſe two altitudes
2d *Hodgſon*, 349. point out the different places, in the heavens, where the ſun is, when he appears upon the ſame azimuth.

PROBLEM VII.

Prob. 7. *Given the azimuth of the ſun, and his declination, together with the hour of the day; to find his altitude.*

EXAMPLE.

The ſun having 19° 39′ north declination, his azimuth at eight hours thirty minutes, in the morning, was found to be ſouth 72° 13′; his altitude is required.

Here are given, the azimuth or angle at the zenith, 72° 13′; the complement of the declination, 70° 21′, being the ſide oppoſite thereto; and the angle at the pole, 52° 30′, equal to the hour from noon; to find the complement of the altitude, which is the ſide oppoſite to the laſt mentioned angle.

This may be reſolved according to the ſecond caſe of oblique ſpherical triangles, *viz.* As the ſine of the angle at the zenith, 72°
2d *Hodgſon*, 256, 158. 13′, to the ſine of the angle at the pole, 52° 30′, ſo is the co-ſine of the declination (19° 39′=) 70° 21′, to the co-ſine of the altitude (38° 19′=) 51° 41′.

The Practice on the QUADRANT.

Take 52° 30′, from the sines, in the compasses; with that extent, enter one foot at 72° 13′ on the sines, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at 70° 21′ on the sines, and, with the other, take the nearest distance to the thread; then, applying this extent to the centre of the sines, it will reach to 51° 41′, the co-sine of 38° 19′, the sun's altitude.

PROBLEM VIII.

The declination and latitude being given, to find the sun's altitude at the hour of six. Prob. 8.

EXAMPLE.

In the latitude of 51° 32′ north, the sun having 19° 39′ north declination; his height at the hour of six is demanded.

The rule, to calculate this, is, As radius, to the sine of the latitude, 51° 32′, so is the sine of the declination, 19° 39′, to the sine of the sun's height, at six, 15° 16′.

The Practice on the QUADRANT.

Take 51° 32′, in the compasses, from the sines; with that extent, enter one foot at radius, and bring the thread to the other; then set one foot of the compasses (discharged of the first extent) to 19° 39′, on the sines, and, with the other take the nearest distance to the thread; this extent, applied to the centre of the sines, will reach to 15° 16′, the altitude of the sun at six, which is the same both morning and afternoon.

PROBLEM IX.

Given the latitude, and hour of the day, the sun being in the equator, to find his height. Prob. 9.

EXAMPLE.

Given the latitude 51° 32′ north, and the hour of the day, *viz.* half an hour after eight in the morning, or after three in the afternoon (which, turned into degrees, is 52° 30′); thence to find his altitude, at that time.

The

2d *Hodgson*, 312. *Gunter*, 104. The proportion will be, As radius, to the co-sine of the hour from noon (52.30′=) 37° 30′, so is the co-sine of the latitude (51° 32′=) 38° 28′, to the sine of the height, at that hour, *viz.* 22° 15′.

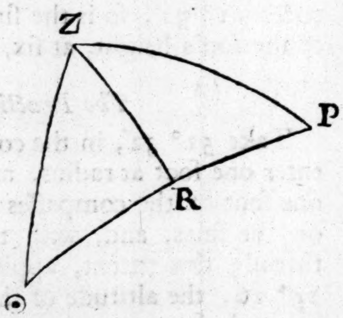
The Practice on the QUADRANT.

Take 37° 30′, from the sines, in the compasses; with that extent enter one foot at radius, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at 38° 28′, on the sines, and, with the other, take the nearest distance to the thread; this extent, applied to the centre of the sines, will reach to 22° 15′, the altitude of the sun at that time and place.

PROBLEM X.

Prob. 10. *Given the latitude of the place, the hour of the day, and the azimuth of the sun; to find his altitude.*

EXAMPLE.



In the oblique spherical triangle Z ⊙ P are given Z P, the complement of the latitude (51° 32′=) 38° 28′, the angle at Z, the azimuth south 72° 13′ west; and the ∠ P, the hour from noon, three hours thirty minutes P M, = 52° 30′; thence to find Z ⊙, the complement of the altitude.

2d *Hodgson*, 369, 184. Let fall the perpendicular ZR, then, (by the eighth case of oblique spherical triangles) As radius, to the sine of the latitude, 51° 32′, so is the tangent of the hour from noon, 52° 30′, to the co-tangent of the angle P Z R, 44° 25′.

And it will hold also thus, As the sine of 51° 32′, to the radius, so is 37° 28′, the co-tangent of 52° 30′, to the tangent of 44° 25′.

The Practice on the QUADRANT.

Take therefore 51° 32′, from the sines, in the compasses; enter one foot (with that extent) at radius, on the tangents, and bring the thread to the other; then take 37° 28′, on the tangents, in the compasses, and, with that extent, entring between the thread and the line of tangents, they will rest at the tangent of 44° 25′.

If from the ſupplement of the azimuth, S. 72° 13′ W. (that is, the azimuth counted from the north) *viz.* 107° 47′, there be taken the angle P Z R, before found, 44° 25′, there will remain the angle ☉ Z R = 63° 22′.

Wherefore, the next proportion will be, As the co-ſine of the angle ☉ Z R (63° 22′ =) 26° 38′, to the co-ſine of the angle P Z R (44° 25′ =) 45° 35′, ſo is the co-tangent of the latitude (51° 32′ =) 38° 28′, to the co-tangent of the altitude (38° 19′ =) 51° 41′. 2d *Hodgſon*, 370.

The Practice on the QUADRANT.

Here, as the co-tangent in the fourth term exceeds the limits of the Quadrant, radius muſt be introduced, and the proportion be divided into the two proportions following; As radius, to the ſine of 45° 35′, one of the middle terms, ſo is the co-tangent of the latitude, 38° 28′, another of the middle terms, to a fourth proportional, which will be the tangent of 29° 34′. *Collins*, 73. 169, 170.

Again, as that tangent, 29° 34′, to radius, ſo is the firſt term, the ſine of 26° 38′, to the tangent of 38° 19′, the altitude required.

The Practice on the QUADRANT *in the firſt part of the proportion.*

Take the ſine of 45° 35′, in the compaſſes; with that extent, enter one foot, at radius, on the tangents, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 38° 28′, on the line of tangents, and, with the other, take the neareſt diſtance to the thread; this extent (applied to the centre of the tangents) will reach to 29° 34′.

Again, in the ſecond part of the proportion. Take 29° 34′, from the tangents, in the compaſſes; enter one foot, at radius, on the tangents, and bring the thread to the other; then take 26° 38′, from the ſines, in the compaſſes, and entring (with that extent) between the thread and the ſcale of tangents, they will reach to 38° 19′, the tangent of the altitude.

PROBLEM XI.

To find the height of the ſun, at all hours, by the verſed ſines on the ſide of the Quadrant; the ſun's place being aſſigned in any point of the zodiac, and the latitude of the place given. The two legs of the triangle, in this caſe, are the complement of the latitude, and the ſun's Prob. 11. *Gunter*, 1ſt part, 184.

diſtance

distance from the elevated pole. The angle intercepted between them, is the hour from the noon, when the altitude is required, the base is the complement of the altitude.

First, find the sum of, and the difference between, the complement of the latitude, and the co declination, or the sun's distance from the elevated pole; count, or note, both the sum, and difference, upon the versed sines, and take in the compasses the distance between them; enter one end, with this distance, at 180°, the end of the scale of versed sines, and bring the thread to the other; then count every hour, upon the scale of versed sines, allowing 15° to an hour, and from those points take the least distance to the thread; these distances, being set off from the aforesaid difference, between the co-latitude and co-declination, forwards upon the versed sines, will give the complement of the altitude, to each several hour, from the meridian.

Note, if you go quite through every 15th degree, or every one of the twelve hours upon the scale, you will go beyond 90°, and those degrees that go beyond 90°, are the profundities or night hours, the sun being in the given degree of the Zodiac; and they are also altitudes of the same hours, when the sun is in the opposite part of the Zodiac; so that one position of the thread, on the Quadrant, will serve to find the altitude, at all the hours in any two opposite points, or signs in the Zodiac.

Note also, that the difference of the two legs (the co-latitude and co-declination) is the complement of the sun's altitude at noon; and the sum of them, being diminished by 90°, is the depth at midnight, or the mid-day altitude of the sun, when he is in the opposite sine or degree.

Example.

Of finding the sun's altitude, at all hours, by the versed sines.

In the latitude of 51° 30′, the sun being in the first point of Taurus, his altitude at every hour of the day, and his profundity or depression at every hour in the night, are required.

78 30 The complement of the latitude given, in this case, is 38° 30′,
38 30 and, as the declination of the sun, when he is in Taurus no de-
grees, is 11° 30′, consequently, his distance from the north pole, or
117 00 his co-declination, is 78° 30′; the sum of these is 117°, and the
40 00 difference is 40°.

First

First then, ſet one foot of the compaſſes at 117°, on the ſcale of verſed ſines, at the ſide of the Quadrant, and extend the other to 40°, on the ſame ſcale; then enter one foot, with that extent, at 180°, the end of the ſame ſcale, and bring the thread to the other; there let it reſt, or keep it faſt. Then count 15°, equal to one hour, from the centre of the verſed ſines; from thence take the neareſt diſtance to the thread; apply one foot of the compaſſes (with this extent) to 40°, and (turning the other forwards) on the ſame line, it will fall on 41° 48′, the complement of the ſun's altitude, at that time, *viz.* at one, or eleven; that is to ſay, at one hour from noon.

Again, count 30° on the ſaid verſed ſines, and apply one foot of the compaſſes to it, with the other take the neareſt diſtance to the thread (remaining in the ſame poſition as before;) then ſet one foot of the compaſſes, with this extent, to 40°, and (turning the other foot forward) it will fall on 46° 48′, which is the complement of the ſun's altitude, at 10 and 2 o'clock, or 2 hours from noon; and, conſequently, his altitude is 43° 12′.

In the ſame manner, taking the neareſt diſtance from 45°, and ſetting one foot of the compaſſes, with that extent, to 40°, you will find the other to fall on 54°, the complement of the ſun's altitude at 9 and 3 o'clock, the altitude itſelf being 36°. Proceeding thus from 60°, the compaſſes will ſhew the complement of the altitude, 62° 29′, and, conſequently, the altitude itſelf, 27° 31′, at the hours of 8 and 4; at 75°, the compaſſes, ſet as above, will give 71° 42′, for the complement of the altitude, and 18° 18′ for the altitude itſelf, at 7 and 5 o'clock; and at 90° or 6 o'clock, the complement of the altitude will be 81°, and the altitude itſelf 9 degrees.

So working on, ſtill in the ſame way, from 105°, on the ſame verſed ſines, the compaſſes will reach a little beyond 90°, *viz.* 90° 06′, for five in the morning and ſeven in the afternoon; from which, if you take 90°, the remainder ſhews how much the ſun is below the horizon, at five in the morning, namely 6 minutes; or it ſhews how high the ſun will be, when it is in the beginning of Scorpio, the oppoſite ſign to Taurus, at ſeven in the morning, and at five in the afternoon; and doing the like from 120°, you will find the compaſſes to ſhew 98° 33′, from which taking 90°, there will remain 8° 33′, for the ſun's profundity or depreſſion, at four in the morning, and eight at night, (the ſun being, as above mentioned, in no degrees in Taurus) or 8° 33′, for the ſun's altitude at eight in the morning, or four in the afternoon, in no degrees of Scorpio.

Again, at 135°, the profundity at 3 and 9 in Taurus, or the altitude at 9 and 3 in Scorpio, will be 105° 58′, from which take 90°, there remains 15° 58′; at 150°, the profundity at 2 and 10, or the altitude at 10 and 2, will be 21° 51′; at 165°, the profundity at 1 and 11, in no degrees of Taurus, or the altitude in no degrees of Scorpio, will be 25° 40′.

Gunter, 187. And, laſtly, whereas the difference of the two legs was found to be 40°, the ſame 40° ſhew the complement of the ſun's altitude, at twelve o'clock, when the ſun is in no degrees of Scorpio.

By this appears the manner of reſolving this propoſition, and how tables of the ſun's altitudes may be made to other ſigns or points of the ecliptic.

Note alſo, that the work may begin with the winter ſigns, and end with the ſummer, that is, it may begin with Scorpio, and end with Taurus. Thus, at the ſun's entrance into Scorpio, his declination is 11° 30′ ſouth, to which add 90°, making together 101° 30′, which is his diſtance from the north pole; to this diſtance add the complement of the latitude, 38° 30′, and it makes 140, the ſum of the two legs, the co-latitude and diſtance from the pole; the difference between them is 63°, the complement of this diſtance is 27°, the altitude for 12 o'clock at noon, in the beginning of Scorpio.

And if you work for the other hours (as in the laſt example) you will find the altitude pointed out, for each hour, in no degrees of Scorpio, until you come to 90°; but when you come beyond 90°, the exceſs ſhews the profundity for the reſt of the hours of the night, in Scorpio, and the altitudes for the anſwerable hours in the beginning of Taurus; and ſo for all other ſigns and parallels of declination.

PROBLEM XII.

Prob. 12. *To find the meridian altitude of the ſun, the latitude and day of the month being given, and the declination found, on the Quadrant, as before directed.*

I. Suppoſe the declination found on the Quadrant to be 23° 29′, or, for a round number, 23° 30′, north, and the complement of the latitude 38° 30′. Theſe added together give the meridian altitude, 62°.

II. Again, if the complement of the latitude be, as before, 38° 30′, and the declination ſouth 23° 30′, the declination, being ſubtracted, gives for the meridian altitude, 15°.

Where-

2

Wherefore, to find the meridian altitude from the winter ſolſtice to the equinox, ſubtract the ſun's declination from the co-latitude: But, to find the ſun's height from the equinox to the ſummer ſtolſtice, to the complement of the latitude, add the declination.

Here obſerve, that the Quadrant, not only reſolves the preceding caſes, relating to the ſun's altitude, according to the rules laid down in the books, but it ſhews it alſo, exactly and expeditiouſly, only, by taking the altitude thro' the two ſights, on the edge of the Quadrant, as at large is ſhewn, in the former part of this diſcourſe.

N. B. Directions are given, for making a table of altitudes, at all hours, in Mr. Collins's Quadrant, page 119.

AMPLITUDE *of the* SUN.

PROBLEM I.

The latitude of the place, and the declination of the Sun, being given, to find his Amplitude; that is, how far he riſes and ſets, from the eaſt and weſt points of the horizon. Prob 1.

EXAMPLE.

In the latitude of 51° 32′ north, the ſun having 19° 39′ declination north, his amplitude is required. 2d *Hodgſon,*

This will be found by the tenth caſe of right angled ſpherical triangles, As the co-ſine of the latitude, 51° 32′, to the radius, ſo is the ſine of the declination, 19° 39′, to the ſine of the amplitude, 32° 44′, as it will appear. 258, 259. 261, 262.

The Practice on the QUADRANT.

Take 38° 28′, the complement of the latitude, from the ſines, in the compaſſes; enter one foot at radius, and bring the thread to the other; then take 19° 39′, in the compaſſes, from the centre of the ſines, and entring (with that extent) between the thread and the ſcale of ſines, they will reſt at 32° 44′, the ſun's amplitude, which is northwardly, becauſe the declination is north, and, conſequently,

 the

the ſun riſes 32° 44′, to the northward of the eaſt point of the horizon, and ſets 32° 44′, to the northward of the weſt point, or, according to the mariner's phraſe, the ſun riſes north eaſt by eaſt, and ſets north weſt by weſt, nearly.

PROBLEM II.

Prob. 2. *Having the latitude of the place, and the diſtance of the ſun from the next equinoctial point; to find the amplitude.*

Example.

Let the latitude of the place be 51° 32′ north, the ſun's place being in Taurus, 28° 36′, or (30° + 28° 36′ =) 58° 36′, from the equinoctial point Aries; thence to find his amplitude.

The rule will be, As the co-ſine of the latitude, 38° 28′, to the ſine of the greateſt declination, 23° 29′, ſo the ſine of the ſun's place, 58° 36′, to the ſine of the amplitude which will be 32° 44′.

The Practice on the Quadrant.

Take 23° 29′ from the ſines, in the compaſſes; with that extent enter one foot at, 38° 28′, the co-latitude, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of their firſt extent) at 58° 36′, on the ſines; thence take the neareſt diſtance to the thread; this extent applied to the centre of the ſines, will reach to 32° 44′, the ſun's amplitude.

Angles *of* Position *and other* Angles.

PROBLEM I.

Prob. 1. *Given the ſun's greateſt declination and his right aſcenſion, to find the angle formed at the ſun by the ecliptic, and the circle of right aſcenſion, or meridian.*

Example.

The ſun's greateſt declination being 23° 29′, and the ſun's right aſcenſion 55° 17′, it is required to find the angle made by the ecliptic, and the circle of right aſcenſion.

And

And this will be found in the ſame manner as in the ſecond Example of the ſeventh Caſe of right angled ſpherical triangles, *viz.* As radius, to the co-ſine of the right aſcenſion, 34° 43′, ſo is the ſine of the greateſt declination, 23° 29′, to the co-ſine of the angle required, 76° 53′, as it will appear. 2d *Hodgſon*, 254, 255, 88.

The Practice on the QUADRANT.

Take 34° 43′, on the ſines, in the compaſſes; with that extent enter one foot at radius on the ſines, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 23° 29′, on the ſines, and with the other take the neareſt diſtance to the thread; this extent, applied to the centre of the ſines, will reach to 13° 07′, the complement of 76° 53′, the angle required.

This problem is of uſe to find out the nonageſimal degree, or higheſt point of the ecliptic, its altitude above the horizon at any given time, whence the true altitude of the ſun, moon, or planet may be inveſtigated, and thence their parallaxes in altitude, right aſcenſion, declination, longitude, and latitude, may be determined, which is neceſſary to be known in calculating ſolar eclipſes, and the moon's tranſits over the fixed ſtars, which are of great uſe in finding the difference of longitude between thoſe places, where they can be obſerved.

PROBLEM II.

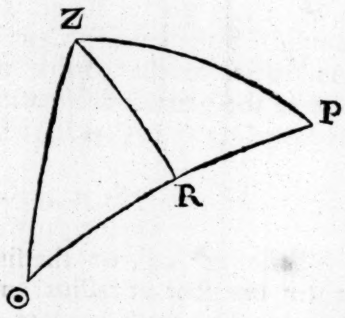
Prob. 2.

Given the latitude of the place, the hour of the day, and azimuth of the ſun; to find the angle formed by the vertical and hour circles, paſſing through the ſun, which is uſually called the Angle of Poſition. 2d *Hodgſon*, 369, 370, 371, 184.

EXAMPLE.

In the latitude of 51° 32′, at 3 hours 30 minutes afternoon, the ſun's azimuth was found to be ſouth 72° 13′ weſt; the angle, formed by the vertical and hour circles, paſſing through the ſun, is required.

In



In the oblique angled ſpherical triangle, ⊙ Z P, are given Z P, the complement of the latitude, 38° 28′, the angle Z, the ſupplement of the azimuth, 72° 13′; and the angle P, the hour from noon, 52° 30′; whence to find the angle of poſition, Z ⊙ P. Let fall the perpendicular Z R, then, by caſe the eighth of oblique angled
2d ſpherical triangles, As the radius, to
Hodgſon, the ſine of the latitude, 51° 32′,
369, 184. ſo is the tangent of the hour from noon, 52° 32′, to the co-tangent of the angle P Z R, 44° 25′.

But the angle P Z ⊙ leſs the angle P Z R = the angle ⊙ Z R, that is, if from the azimuth (counted from the north), 07° 47′, be taken the angle P Z R, 44° 25′, there will remain the angle ⊙ Z R, equal to 63° 22′.

2d *Hodgſon*, 371. Again (by Caſe 7.) As the ſine of the angle P Z R, 44° 25′, to the ſine of the angle ⊙ Z R, 63° 22′, ſo is the co-ſine of the hour from noon (52° 32′=) 37 28′, to the co-ſine of the angle of poſition Z ⊙ P, 38° 58′.

The Practice on the QUADRANT, *in the firſt proportion*, viz.

As radius, to the ſine of the latitude, 51° 32′, ſo is the tangent of 52° 32′, to the co-tangent of the ∠ P Z R. But, becauſe the tangent of 52° 32′, exceeds the bounds of the Quadrant, therefore change the firſt and ſecond terms, which will infer a change of the tangent in the third term into the co-tangent, and the laſt term into a tangent, and then it will ſtand thus, As the ſine of 51° 32′, to the radius, ſo is the co-tangent of (52° 32′=) 37° 38′, to the tangent of 44° 25′, as it will appear.

Therefore, take 51° 32′, from the ſines, in the compaſſes; with that extent enter one foot at radius on the tangents, and bring the thread to the other; then take 37° 28′, on the tangents, from the centre, and, with that extent, entring between the thread and the line of tangents, it will reſt at 44° 25′, the angle P Z R. Then, as before, the angle P Z ⊙, 107° 47′, leſſened by P Z R, 44° 25′, is equal to ⊙ Z R, 63° 22′.

107	47
44	25
63	22

Therefore, to perform the ſecond proportion, by the Quadrant, Take 44° 25′, from the ſines, in the compaſſes; with that extent enter one foot at 63° 22′, on the ſines, and bring the thread to the other;

other; then take 37° 28′, on the ſines, in the compaſſes, from the centre, and with that extent entring them between the thread and the ſcale of ſines, they will reſt at 52° 32′, the complement of 38° 58′, the angle of poſition.

PROBLEM III.

The latitude of the place, and altitude of the ſun, and the hour from noon given; to find the Angle of Poſition. Prob. 3.

EXAMPLE.

Given the latitude of the place, 51° 32′, the altitude of the ſun 38° 19′, and the hour from noon 52° 30′; thence to find the angle required, *Taylor's* Theſaurus, 109.

The rule will be, As the co-ſine of the ſun's altitude (38° 19′=) 51° 41′, to the ſine of the hour from noon, 52° 30′, ſo is the co-ſine of the latitude (51° 32′=) 38° 28′, to the ſine of the angle of poſition, 38° 58′.

The Practice on the QUADRANT.

Take 51° 41′, from the ſines, in the compaſſes; with that extent enter one foot at 52° 30′ on the ſines, and bring the thread to the other; then take 38° 28′, in the compaſſes, from the centre of the ſines, and entring them at that extent, between the thread and the ſcale of ſines, they will reſt at 38° 58′, the angle of poſition.

PROBLEM IV.

MERIDIAN ANGLE.

Given the ſun's greateſt declination, 23° 29′, *and his place in the ecliptic,* 30° 00′; *thence to find the meridian angle, that is, the angle made with the meridian by the ecliptic, at the ſun's place.* Prob. 4.

The rule is, As radius, to the co-ſine of the ſun's place (30°=) 60° 00′, ſo is the tangent of the ſun's greateſt declination, 23° 29′, to the co-tangent of the fourth proportional required, *viz.* 20° 38′, whoſe complement is 69° 22′, the angle ſought. *Partridges* Double Scale, 111.

The Practice on the QUADRANT.

Take 60° from the ſines, in the compaſſes; with that extent enter one foot at radius, on the tangents, and bring the thread to the other;

other; then enter one foot of the compasses (discharged of the first extent) at 23° 29′ on the tangents, and, with the other, take the nearest distance to the thread; this extent, applied to the centre of the tangents, will reach to 20° 38′, whose complement is 69° 22′, the angle sought.

PROBLEM V.

Prob. 5. *To find the angle of the meridian with the horizon.*

The altitude of the equator, or latitude of the place, and the sun's declination, being given, thence to find the angle which the meridian, passing thro' the sun, makes with the horizon, at the time of the sun's rising or setting.

EXAMPLE.

Oughtred's Proportions, 94. Let the altitude of the equator be 51° 30′, the sun's declination 22°, and let the above angle be required, then it will be, As 68°, the co-sine of the declination, to the radius, so is the sine, 51° 30′, the altitude of the equator, to the angle required, which will be 57° 34′.

The Practice on the QUADRANT.

Take 68° from the sines, in the compasses; with that extent enter one foot at radius on the sines, and bring the thread to the other; then take 51° 30′ from the sines, in the compasses, and entring, with that extent, between the thread and the line of sines, it will rest at 57° 34′, the angle of the meridian with the horizon.

RIGHT ASCENSION.

PROBLEM I.

Prob. 1. *Given the sun's greatest and present declination; to find his right ascension.*

EXAMPLE.

Given his greatest declination — — — 23° 29′.
His present declination north and increasing — 19° 39′,
To find the sun's right ascension.

This may be performed by Case the ninth of right-angled spherical triangles. As the tangent of the greatest declination, 23° 29′; to the tangent of the present declination, 19° 39′; so is the radius, to the sine of the right ascension, 55° 17′; as it will appear. Now, because the declination is north and increasing, the sun is in the first Quadrant of the eclyptic, and consequently the arc 55° 17′ is the right ascension from Aries, without any addition. 2d *Hodgson*, 251, 96.

The Practice on the Quadrant.

Take 19° 39′ from the tangents in the compasses, with that extent, enter one foot at 23° 29′ on the tangents, and bring the thread to the other; then enter one foot of the compasses, discharged of the first extent, at radius, on the tangents, and with the other take the nearest distance to the thread; this extent applied to the centre of the sines, will reach to 55° 17′, the right ascension required. But this is much more easily done, by inspection, on the Quadrant, than by the preceding rule; for here you need only to lay the thread to the day of the month, or to the declination in the circle of declination; and the thread gives the right ascension in the limb of the Quadrant, remembring to add 90°, if the right ascension exceeds the first Quadrant; 90° more, if it exceeds the second; and 90° more, if it exceeds the third. See, for this, what is said before concerning it, and also 2d *Hodgson*, 248, 249.

PROBLEM II.

The sun's place and greatest declination given, to find his right ascension.

Example.

On the 7th of *May* at noon, the sun being in *Taurus* 27° 34′, his right ascension is required. The proportion will be; As radius to the co-sine of the sun's greatest declination, (23° 29′ =) 66° 31′; so is the tangent of the sun's distance from the first point of *Aries*, 57° 34′; to the tangent of the right ascension 55° 17′: Or, changing the first and second terms to bring the tangents within the limits of the Quadrant, the proportion may stand thus: As the cosine of the greatest declination, (23° 29′ =) 66° 31′, to radius; so is the co-tangent of (57° 34′ =) 32° 26′, to the co-tangent of (55° 17′ =) 34° 43′.

27	34
30	
57	34

2d *Hodgson*, 246, 249, 77.

The Practice on the Quadrant.

Take 66° 31′ from the sines, in the compasses; with that extent enter one foot at radius on the tangents, and bring the thread to the other;

other; then take 32° 6′, from the centre of the tangents, in the compaſſes, and entering with that extent between the thread and the ſcale of tangents, they will reſt at 34° 43′; whoſe complement is 55° 17′, the ſun's right aſcenſion required.

Here, again, the Quadrant gives the anſwer, much eaſier than by following the preceding rule: for if you count 27°, and (by the eye) an half more from *Taurus*, and lay the ſtring over there, or on the 7th of *May*, it will give you, 55° 17′, on the limb of the Quadrant, for the right aſcenſion.

OBLIQUE ASCENSION *and* DESCENSION.

IN order to the ſolution of Problems relating to the ſun's oblique aſcenſion, and deſcenſion, or the length of the diurnal, and nocturnal arcs, and thence the time of his riſing and ſetting, theſe things are to be premiſed: Firſt, the ſun's nocturnal arc is that ſpace in the heavens, which the ſun runs through, from the time of his ſetting to the time of his riſing: For inſtance, when he riſeth (ſuppoſe at four o'clock in the morning) that time doubled is eight hours, the length of his nocturnal arc; and the diurnal arc is always equal to the double of the time of the ſun's ſetting: Secondly, the times of the ſun's riſing and ſetting, are always complements of each other, to twelve hours; and the lengths of the day and night are complements of each other, to twenty-four hours.

1ſt *Leadbetter*, 36.

PROBLEM I.

Prob. 1. *The latitude of the place being given, with the ſun's declination; thence to find his oblique aſcenſion, or the degree of the equator which riſes with him.*

EXAMPLE.

2d *Hodgſon*, 270, 104.

In the latitude 51° 32′, the ſun having 19° 39′ of north declination; his oblique aſcenſion is required; to perform which, the rule is, according to the twelfth caſe of right-angled ſpherical triangles, As radius, to the tangent of the latitude, 51° 32′; ſo is the tangent of the declination, 19° 39′; to the coſine (in this caſe) of the ſemi-nocturnal arc (63° 17′ =) 26° 43′; which 63° 17′, being reduced into time, gives 4 hours 13 minutes, for the time of the ſun's

sun's rising; and, subtracted from 12 hours, gives 7 hours 47 minutes for his setting.

Again, the same semi-nocturnal arc, 4 hours 13 minutes, being doubled, gives 8 hours 26 minutes, for the length of the night; and this, taken from 24 hours, gives 15 hours 34 minutes, for the length of the day.

Again, the complement of the semi-nocturnal arc ($63^\circ\ 17' =$) $26^\circ\ 43'$, subtracted from the sun's right ascension, $55^\circ\ 17'$, found by the preceding problem; will give, for the oblique ascension, $28^\circ\ 34'$; and the complement of the semi-nocturnal arc, $26^\circ\ 43'$, added to the said $55^\circ\ 17'$, gives for the oblique descension, $82^\circ\ 00'$.

The Practice, on the QUADRANT, *of the preceding Problem.*

Here, to avoid the tangent of $51^\circ\ 32'$, which exceeds the limits of the Quadrant, the analogy (changing the places of the two first terms) will be thus, As the co-tangent of the latitude ($51^\circ\ 32' =$) 2d *Hodgson*, 270.
$38^\circ\ 28'$, to the radius; so is the tangent of the declination, $19^\circ\ 39'$; to the co-sine (in this case) of the semi-noctural arc ($63^\circ\ 17' =$) $26^\circ\ 43'$. Take $38^\circ\ 28'$ from the tangents in the compasses, with that extent, as the last proportional is to be found on the sines, enter one foot at radius on the sines, and bring the thread to the other; then take, from the centre of the tangents, $19^\circ\ 39'$, and entering, with this extent, between the thread, and the line of sines, they will rest at $26^\circ\ 43'$, the complement, of $63^\circ\ 17'$, the semi-nocturnal arc required.

Here, again, the Quadrant eases the trouble of calculation; for if you lay the thread over the 7th of *May*, in the upper circles of months, or (which is the same thing) over the twenty-seventh degree of *Taurus*, it will cut the limb at $55^\circ\ 17'$ for his right ascension.

Again, laying the thread over the 7th of *May*, in the lower circular arcs of months, if you count the minutes, on the straight line of hours, from six hours, to four hours less thirteen minutes; there are in all 107 minutes, and something more, which turned into degrees, gives $26^\circ\ 43'$; and this, taken from $55^\circ\ 17'$, the sun's right ascension, gives for the oblique ascension $28^\circ\ 34'$, as before; and added to it, gives 82° for the oblique descension.

PROBLEM II.

Prob. 2. *Given the right aſcenſion, and aſcenſional difference, thence to find the oblique aſcenſion and deſcenſion.*

Rule.

1ſt *Leadbetter*, 111. In north latitude, if the declination be north, ſubtract the aſcenſional difference from the right aſcenſion, and this gives the oblique aſcenſion; and add the aſcenſional difference to the right aſcenſion, for the oblique deſcenſion.

If the declination is ſouth, add the aſcenſional difference to the right aſcenſion, and this gives the oblique aſcenſion; and ſubtract the aſcenſional difference from the right aſcenſion for the oblique deſcenſion. In ſouth latitude, juſt the contrary.

Example.

Let the right aſcenſion of the ſun be 47° 29′, and the aſcenſional difference 23° 46′, in the latitude 51° 32′ north, when the ſun has north declination.

From the right aſcenſion — — — — — —	47	29
Subtract the aſcenſional difference — — —	23	46
And there remains the oblique aſcenſion — —	23	43
To the aſcenſional difference — — — —	23	46
Add the right aſcenſion — — — — —	47	29
And the oblique deſcenſion will be — — —	71	15

1ſt *Leadbetter*, 110, 111. *Note*, That in north latitudes, the time of the ſun's riſing, when he is in the northern ſines, is the time of his ſetting, when in the ſouthern ſigns, and the contrary; for inſtance, If the ſun riſes at three hours 47′ 23″ 44‴ after midnight, when he is in the tropic of *Cancer*, then the ſame number of hours, *&c.* will be the time of his ſetting after noon, when he is in the tropic of *Capricorn*.

Note,

Note alſo, that the ſun's declination is ſuppoſed to be unalterable for one day, and therefore in the projection of the ſphere, it is called a parallel of declination, and is ſo drawn; but this, ſtrictly ſpeaking, is not ſo; for they are not parallel, but ſpiral lines, which the ſun deſcribes from tropic to tropic.

ASCENSIONAL DIFFERENCE.

THIS is the difference between the right and oblique aſcenſion; or the ſpace of time, which the ſun riſeth and ſetteth, before or after ſix; that is, it is ever equal to the exceſs, or defect, of the ſemi-ſolar day, above or under ſix hours. 2d *Hodgſon*, 265.

PROBLEM I.

Given the latitude of the place 51° 32′ *and the declination* 19° 39′ *north, to find the aſcenſional difference.* Prob. 1.

The rule is (according to the ninth caſe of right-angled ſpherical triangles) As the co-tangent of the latitude (51° 32′ =) 38° 28′; to the radius; ſo is the tangent of the declination, 19° 39′; to the ſine of the aſcenſional difference, 26° 43′. 2d *Hodgſon*, 264, 96.

The Practice on the QUADRANT.

Take 38° 28′ from the tangents in the compaſſes; with that extent enter one foot at radius on the ſines (as the laſt proportional is to be found there) and bring the thread to the other; then take 19° 39′ from the centre of the tangents, in the compaſſes, and, with that extent, entring them between the thread and the line of ſines, they will reſt at 26° 43′ the aſcenſional difference.

But this caſe may be reſolved much eaſier, by the Quadrant, in the following manner: Put the ſtring over 19° 39′ in the circular line of the ſun's declination, and it will cut over head at the ſeventh of *May*, and underneath in the limb it will cut 55° for the ſun's right aſcenſion. In the next place, ſet the thread over the ſeventh of *May*, in the lower arcs of months, and it gives the time of the ſun's riſing, in the ſtraight line of hours over head, at four hours leſs 13 minutes; and the time of his ſetting at ſeven hours more

47 minutes; and counting, in the ſame hour line, the minutes, between ſix, and ſeven hours 47′, you will have 107 minutes; which being turned into degrees, gives, for the aſcenſional difference, 26° 43′, as before; and you have, at the ſame time, obtained the ſun's right aſcenſion, 55°, and his time of riſing and ſetting, as above.

Prob. 2.

PROBLEM II.

2d Hodgſon, 289.

The ſun's amplitude, and preſent declination being given; to find his aſcenſional difference.

Example.

Given the amplitude 32° 44′ northerly, in the morning, and the ſun's declination 19° 39′ N; thence to find his aſcenſional difference; then (by the fourteenth caſe of right-angled ſpherical triangles.) As the co-ſine of the declination (19° 39′ =) 70° 21′; to the co ſine of the amplitude (32° 44′ =) 57° 16′; ſo is the radius; to the coſine of the aſcenſional difference 26° 43′.

P. 107, 108.

The Practice on the Quadrant.

Take 57° 16′ from the ſines in the compaſſes, enter one foot with that extent at 70° 21′, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at radius, and thence with the other take the neareſt diſtance to the thread: This extent, applied to the centre of the ſines, will reach to 63° 17′ the complement of the ſun's aſcenſional difference 26° 43′: Which aſcenſional difference, converted into time, and ſubtracted from ſix hours, will give four hours leſs 13 minutes, as before, for the time of the ſun's riſing, or hour of the day; the latitude and declination being the ſame way.

But this is reſolved much eaſier, by the Quadrant; thus, the declination being 19° 39′, lay the thread over it, and the day is pointed out by it, over head, to be the ſeventh of *May*; then lay the thread over the lower arcs of months, at the ſeventh of *May*, and it cuts the hour, over head, at four hours leſs 13 minutes, as before.

3

Azi-

AZIMUTH *of the* SUN.

PROBLEM I.

The latitude of the place, and declination of the ſun being given; to find his azimuth at the hour of ſix. Prob. 1.

Example.

In the latitude 51° 32′, the ſun having 19° 39′ north declination; his azimuth at ſix is required. Now (by the ſecond example of the fourth caſe of right-angled ſpherical triangles) As radius to the co-ſine of the latitude (51° 32′ =) 38° 28′; ſo is the tangent of the declination, 19° 39′; to the co-tangent of the azimuth from the meridian 77° 28′, = the tangent of 12° 32′. 2d *Hodgſon*, 295, 78.

The Practice on the Quadrant.

Take 38° 28′, the complement of 51° 32′, from the ſines, in the compaſſes; with that extent enter one foot at radius on the tangents, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 19° 39′ on the line of tangents, and with the other take the neareſt diſtance to the thread; this extent, applied to the centre of the tangents, will reach to 12° 32′, the complement of 77° 28′ the azimuth from the meridian. Which azimuth muſt always be counted from the viſible pole, that is, if the place be in north latitude, it muſt be reckoned from the north, but if the place be in ſouth latitude, then from the ſouth; and conſequently, in the preſent caſe, if it be in the morning, the azimuth is north 77° 28′ eaſt, and the diſtance from the eaſt 12° 32′; But if it be in the afternoon it is north 77° 28′, weſt, or, according to the ſeaman's phraſe, the ſun is eaſt by north, nearly, at ſix in the morning; and weſt by north, nearly at ſix in the afternoon.

PROBLEM II.

Prob. 2. *Given the ſun's declination, and his altitude at the hour of ſix; to find his azimuth.*

EXAMPLE.

The ſun having 19° 39′ north declination, and his altitude at the hour of ſix being found to be 15° 16′; his azimuth is demanded. Then (by the fourteenth caſe of right-angled ſpherical triangles) As the coſine of the altitude (15° 16′ =) 74° 44′; to the radius; ſo is the co-ſine of the declination (19° 39′ =) 70° 21′; to the ſine of the azimuth, from the meridian, 77° 28′.

2d *Hodgſon*, 297, 111.

The Practice on the QUADRANT.

Take 74° 44′, the complement of the altitude, from the ſines, in the compaſſes, with that extent enter one foot at radius, and bring the thread to the other; then take 70° 21′ the complement of the declination in the compaſſes, from the centre of the ſines, and (entring with that extent) between the thread, and the ſines, they will reſt at 77° 28′, the ſun's azimuth from the meridian.

PROBLEM III.

Prob. 3. *The ſun being upon the equator, the latitude of the place, and hour of the day being given, to find the azimuth.*

EXAMPLE.

In the latitude of 51° 32′ north, the ſun being in the equator at half an hour paſt three in the afternoon, or half an hour paſt eight in the morning, the ſun's azimuth is required. To find which the proportion will be, according to the fourth caſe of right angled ſpherical triangles; As radius, to the ſine of the latitude, 51° 32′; ſo is the co tangent of the hour from noon, 52° 30′; to the co-tangent of the azimuth from noon 59° 00′; as it will appear.

2d *Hodgſon*, 312, 313, 76.

The Practice on the QUADRANT.

Take 51° 32′ from the ſines in the compaſſes, with that extent enter one foot at radius on the tangents, and bring the thread to the other;

other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 37° 30′ the complement of the hour from noon on the tangents, and from thence with the other take the neareſt diſtance to the thread; this extent, applied to the centre of the tangents, will reach to 31° 00′ the complement of the azimuth ſought.

PROBLEM IV.

The ſun being in the equator, there is given the latitude of the place, and the altitude of the ſun, to find his azimuth. Prob. 4.

EXAMPLE.

In the latitude of 51° 32′, the ſun being in the equator, and his altitude found by obſervation to be 22° 15′; his azimuth is required. The proportion will be (by the ninth caſe of right-angled ſpherical triangles) As the co-tangent of the latitude, 51° 32′, to the radius; ſo is the tangent of the altitude, 22° 15′; to the co-ſine of the azimuth from the meridian 59° 00′. 2d *Hodgſon*, 315, 94.

The Practice on the QUADRANT.

Take 38° 28′ the complement of the latitude from the tangents in the compaſſes, with that extent (as the laſt proportional is to be taken on the ſines) enter one foot at radius, on the ſines, and bring the thread to the other; then take the altitude 22° 15′ in the compaſſes from the centre of the tangents, and entering the compaſſes with that extent between the thread and the line of ſines, they will reſt at 31° 00′ the complement of the azimuth required.

PROBLEM V.

The ſun being in the equator, there is given his altitude, and the hour of the day, to find his azimuth. Prob. 5.

EXAMPLE.

The ſun being in the equator, at thirty minutes paſt eight in the morning, his altitude was found by obſervation to be 22° 15′; his azimuth is required. The proportion will be (by caſe the fourteenth of right-angled ſpherical triangles) As the co-ſine of the altitude, 22° 15′; to the radius; ſo is the ſine of the hour from noon, 52° 30′; to the ſine of the azimuth from the meridian, 59° 00′. 2d *Hodgſon*, 318, 110.

The Practice on the QUADRANT.

Take 67° 45′ the complement of the altitude from the ſines in the compaſſes, with that extent enter one foot at radius on the ſines, and bring the thread to the other; then ſet one foot of the compaſſes (diſcharged of the firſt extent) at the centre of the ſines, and extend the other to 52° 30′ on the ſame line; with this extent, enter the compaſſes between the thread and the ſcale of ſines, and they will reſt at, 59° 00′, the azimuth required.

PROBLEM VI.

Prob. 6. *In the latitude* 51° 20′ *north, on the* 27*th of* Auguſt 1732, *the ſun then having* 5° 56′ *north declination; it is required to find the ſun's azimuth, at nine hours four minutes thirty-two ſeconds in the morning.*

The Com. of
9ʰ 4′ 32″ is
2 55 28

2ʰ = 30°
55 = 13 : 45
28 = 7

43 : 52

The azimuth may be here found, as in Prob. 4. of finding altitudes, page 48; that is to ſay, the proportion will be, by the tenth caſe of oblique ſpherical triangles, As the radius, to the co-ſine of the angle from noon, 43° 52′; ſo is the co-tangent of the declination, 5° 56′; to the tangent of a fourth arc, 81° 48′.

The Practice on the QUADRANT, *of the above proportion.*

Here, becauſe the co-tangent of the declination, 84° 4′, exceeds the bounds of the Quadrant, the two firſt terms muſt change places, which will infer a change of that co-tangent into a tangent, and of the laſt proportional into a co-tangent; and then the analogy will be, As the co-ſine of the hour, 43° 52′; to the radius; ſo is the tangent of, 5° 56′; to the co-tangent of, 81° 48′, the fourth arc.

Therefore take 46° 8′, the complement of the hour, from the ſines, in the compaſſes, with that extent enter one foot at radius on the tangents, and bring the thread to the other; then take 5° 56′ from the centre of the tangents in the compaſſes, and with that extent, entering between the thread and the line of tangents, they will reſt at 8° 12′ on the tangents; whoſe complement is 81° 48′ the fourth arc; then from the fourth arc, 81° 48′, ſubtract the co-latitude, 38° 40′; and you have a fifth arc, 43° 8′.

2d *Hodgſon*, 324. Whence the ſecond proportion will be, As the ſine of the fifth arc, 43° 8′; to the ſine of the fourth arc, 81° 48′; ſo is the tangent of the hour from noon, 43° 52′; to the tangent of the azimuth, 54° 18′.

I But

But to bring the tangent of 54° : 18′ in this proportion within the bounds of the Quadrant, there muſt be ſolved two proportions, introducing radius as the firſt term in the firſt of them, and they will ſtand thus: As radius, to the ſine of one of the middle terms beforementioned, *viz.* 81° 48′; ſo is the tangent of the other middle term, 43° 52′; to the tangent of a ſixth arc, 43° 34′: Again, as the tangent of the ſixth arc, 43° 34′ to the radius; ſo is the ſine of the fifth arc, 43° 8′; to the tangent of 35° 42′; whoſe complement is, 54° 18′, the required azimuth from noon.

The Practice on the QUADRANT *of the firſt of the two laſt-mentioned proportions.*

Take 81° 48′ from the ſines in the compaſſes, with that extent enter one foot at radius on the tangents, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at the tangent of 43° 52′, and with the other take the neareſt diſtance to the thread; this extent, applied to the centre of the tangents, will reach to 43° 34′ the ſixth arc.

Again, for the ſecond of thoſe proportions, Take the tangent of the ſixth arc, 43° 34′, in the compaſſes, with that extent enter one foot at radius on the tangents, and bring the thread to the other; then ſet one foot of the compaſſes (diſcharged of the firſt extent) on the centre of the ſines, and extend the other to the ſine of 43° 8′; enter with this extent between the thread and the tangents, and they will reſt at the tangent of 35° 42′; whoſe complement, 54° 18′, is the azimuth required.

PROBLEM VII.

Prob. 7. 2d Hodgſon, 333.

Given the latitude of the place and the declination of the ſun, the one north the other ſouth; together with his altitude; to find his azimuth.

EXAMPLE.

In the latitude 51° 32′ north, the declination of the ſun being 19° 39′ ſouth, and his altitude 15° 30′; his azimuth is required.

Here, becauſe the latitude is north, and the declination ſouth, therefore to 90°, or a Quadrant, add the declination 19° 39′, and the ſum 109° : 39′ will be the ſun's diſtance from the north or viſible pole. Then

To the complement of the latitude — — — —	38°	28′
Add the complement of the altitude — — — —	74	30
And the distance of the sun from the north or visible pole	109	39
The sum is — — — — — — — — —	222	37
And the half thereof — — — — — — — —	111	18
2d *Hodgson*, 334. From this half sum subtract the sun's distance from the pole	109	39
And the difference will be — — — — — — —	1	39

And reserving this difference, the proportion will be, As radius; to the co-sine of the latitude, 51° 32′; so is the co-sine of the altitude, 15° 30′; to a fourth sine, 36° 50′, which may be found in the manner already, sufficiently described.

Again, as this fourth sine, 36° 50′; to the sine of half the sum, 111° 18′, whose supplement is 68° 42′; so is the sine of the difference before reserved, 1° 39′; to another sine 2° 34′, as will be found in the common way; to which add radius, (says Mr. *Hodgson*, 2d vol. 334) and half that sum will be the co-sine of 77° 46′; which being doubled becomes 155° 32′ the azimuth from the north: But to add radius without recourse to logarithms, as in Mr. *Hodgson*, is not, as I know of, practicable; but this difficulty may be removed by using the versed sines, the manner of performing which by the *Gunter's* Scale, Mr. *Hodgson* describes in vol. 1. page 133; which may be applied, to the Quadrant, thus: Take the above extent 2° : 34′, from the centre of the sines, in the compasses, and transfer it to 180°, on the versed sines, at the side of the Quadrant; and it will reach backwards on those sines to 155° : 32′, the azimuth from the north.

PROBLEM VIII.

Prob. 8. *Given the latitude of the place, and declination of the sun, both north; together with his altitude; to find his azimuth.*

EXAMPLE.

In the latitude 51° 32′ north; the sun having 19° 39′ north declination, his altitude was found, by the Quadrant, to be 38° 19′; his azimuth is required.

The most convenient method of performing this by the Quadrant will be as follows:

To

To the complement of the altitude —	38° : 19′, *viz.*	51° : 41′
Add the complement of the latitude —	51 : 32, *viz.*	38 : 28
The ſum will be — — — — — — —		90 : 9
Alſo from the complement of the altitude, *viz.* —		51 : 41
Take the complement of the latitude, *viz.* — —		38 : 28
The difference will be — — — — — —		13 : 13

Now take, in the compaſſes, the diſtance between the above found ſum and difference, *viz.* 90° 9′ and 13° 13′ on the verſed ſines at the ſide of the Quadrant, and enter one foot with this extent at 180, the end of thoſe ſines, and bring the thread to the other; then take in the compaſſes (diſcharged of their firſt extent) the above difference between 13° 13′ and the complement of the declination, *viz.* 70° : 21′, on the ſame ſcale of verſed ſines, and entering, with that extent, between the thread and ſcale of verſed ſines; they will reſt at 107° 48′, the required azimuth from the north.

PROBLEM IX.

Given the altitude of the ſun, and his preſent declination, together with the hour of the day; to find the azimuth. Prob. 9.

EXAMPLE.

The ſun having 19° 39′ north declination, at three hours thirty minutes in the afternoon, his altitude was found to be 38° 19′, his azimuth is required. 2d *Hodgſon*, 342, 344, 156.

The proportion will be (by the firſt caſe of oblique ſpherical triangles) as the ſine of 51° 41′ the complement of the altitude 38° 19′, to the co-ſine of the declination 19° 39′; ſo is the ſine of the hour from noon 52° 30′ to the ſine of the azimuth from the ſouth, 72° 13′.

The Practice on the QUADRANT.

Take 51° 41′ the complement of the altitude from the ſines in the compaſſes, with that extent enter one foot at 70° 21′, the complement of the declination, and bring the thread to the other; then

take,

take, in the compaſſes, 52° 30′ from the ſines, and entering that extent, between the thread and the ſcale of ſines, they will reſt at 72° 13′, the azimuth from the ſouth.

DAYS *and* NIGHTS.

PROBLEM I.

To find the beginning, duration, and end of the longeſt day, and longeſt night; Suppoſe, for inſtance, at the north cape, in the latitude of 71° 25′ north; the complement of which latitude is 18° 35′.

2d *Hodgſon*, 252, 273.

TO determine theſe, the proportion will be, As the ſine of the ſun's greateſt declination, 23° 29′; to radius, 90°; ſo is the ſine of the complement of the latitude, 18° 35′; to the ſine of the ſun's longitude, reckoned from the neareſt equinoctial point, *viz.* 53° 6′.

When the ſun's declination is north, and increaſing, then from, 53° 6′, the above found longitude, take one ſign or 30′ and there remains 23° 6′ for the ſun's place in *Taurus*; in which place he happens to be on the third of *May*; which is the beginning of the longeſt day, in this caſe; but when the declination is north, and decreaſing; the ſun will be in *Leo* 6° 54′; and this happens upon the 19th of *July*, at which time the longeſt day ends; and conſequently the longeſt day conſiſts of 77 natural days.

When the ſun's declination is ſouth, and increaſing; his place will be in *Scorpio*, 23° 6′; which happens upon the fourth of *November*, when the longeſt night begins: And, laſtly, when the ſun's declination is ſouth, and decreaſing, his place will be in *Aquarius*, 6° 54′; and this happens on the 16th of *January*, when the longeſt night ends; and conſequently the length of the longeſt night conſiſts of 73 days; which is four days ſhorter than the length of the longeſt day above found.

The

The Practice on the QUADRANT.

What is told above is much easier shewn by the Quadrant; if it be observed, that the circle of declination thereon begins on the left hand of the Quadrant and proceeds to 23° 29′, increasing; consequently (if we return from the right to the left hand) its decrease will be represented. In conformity to this, the lowest annulus of the upper circles of months, begins at the left hand with the 10th of *March*, when the sun enters *Aries*, thence it goes to *April* and *May*, and so to the 10th of *June*, during which time the declination is north increasing. The next higher annulus begins at the right hand and goes on to *July* and *August* on the left, and to the 12th of *September*, the declination being north decreasing. The third annulus begins at the left hand with the 13th of *September*, and proceeds to the 10th of *December*, the declination being south increasing, and the highest annulus returns, from the right to the left, back to the 9th of *March* the declination being south decreasing. Now, in the present case, the longitude of the sun being above found to be in *Taurus* 23° 6′, lay the thread over *Taurus*, 23° 6′, and it will cut the line of declination at 18° 35′; in the lower annulus of months, progressive or increasing, it lays over the third of *May*. Proceed with your eye to the second annulus of months, when the declination is decreasing, and you will find it to lie over the 19th of *July*, and over 6° 54′ of *Leo*; then carrying your eye to the third annulus again, increasing, and you will find the thread lies over the fourth of *November*, and over 23° : 6′, the sun's place in *Scorpio*.

And lastly, the sun's declination being south decreasing, you will find that the thread rests over *Aquarius* 6° 54′, and over the 16th day of *January*. All which is seen by one position of the thread, in the clearest and exactest manner possible, agreeing entirely with what is laid down by Mr. *Hodgson* as above; and thus having found, as above, when the longest day begins, you have the rest without farther trouble or consideration.

PROBLEM II.

To find what two days are of equal length. Prob. 2.

It is manifest that two days are of equal length, if the sun rises in both of them at the same time; therefore lay the thread to any day in the lower arcs of months; and whatever other day the thread lies over, the same is of equal length with the former.

Ex-

EXAMPLE.

The first of *April* and the 21st of *August* are days of equal length, the sun rising and setting on those days at the same time; observing *Brown* on the Quadrant, 447. only, that in these arcs of months, the upper line shews those times when the days are increasing in length; and the lower, when they are decreasing.

DECLINATION *of the* SUN.

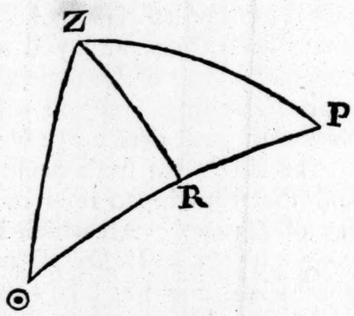
PROBLEM I.

Given the latitude of the place, the sun's altitude, and the hour from noon, to find the declination.

EXAMPLE.

IN the oblique angled spherical triangle, Z P $\odot$, are given; Z P, the complement of the latitude, $38° 28'$, Z $\odot$, the complement of the altitude, $51° 41'$; and the angle at P, the hour from noon, $= 52° 30'$; thence to find P $\odot$, the complement of the declination; here (having let fall the perpendicular Z R,) the proportion will be (by the 1st example of the third case of oblique angled spherical triangles)

2d *Hodgson*, 262, 161.



As the radius; to the co-sine of the angle P, the hour from noon, $52° : 30'$; so is the tangent of Z P, the complement of the latitude, $38° 28'$; to the tangent of a fourth arc, $PR = 25° 49'$: Again, as the sine of the latitude, $51° 32'$; to the sine of the altitude from the horizon, $38° 19'$; so is the co-sine of the fourth arc P R, $25° 49'$; to the co-sine of a fifth arc, $\odot$ R, $44° 32'$.

The

The Practice on the QUADRANT, *in the first proportion.*

Take 37° 30′, the complement of the hour, from the ſines in the compaſſes, enter, with that extent, one foot, at radius on the tangents, bring the thread to the other foot; then entering one foot of the compaſſes (diſcharged of the firſt extent) at the tangent of 38° 28′, with the other take the neareſt diſtance to the thread: This extent applied to the centre of the tangents, will reach to 25° 49′ the fourth arc.

The Practice on the QUADRANT, *in the ſecond proportion.*

Take 38° 19′, the ſun's altitude, from the ſines in the compaſſes, enter one foot (with that extent) at the ſine of 51° 32′, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 64° : 11′, the complement of the fourth arc 25° 49′, on the ſines, and with the other take the neareſt diſtance to the thread: This extent, applied to the centre of the ſines, will reach to 45° 28′ the complement of the fifth arc 44° 32′: But $PR + R\odot = P\odot$, that is the fourth arc, 25° 49′, added to the fifth arc, 44° 32′, gives 70° : 21′, for $P\odot$, the complement of the declination; which, taken from 90°, leaves 19° 39′ for the ſun's declination, north.

PROBLEM II.

Given the latitude of the place, the ſun's azimuth and hour of the day, to find his declination. Prob. 2.

EXAMPLE.

In the latitude of 51° 32′ north, the ſun's azimuth was found to be ſouth 72° : 13′ weſt, at three hours thirty minutes after noon; his declination is required.

By proceeding as in Prob. X. page 54, the altitude of the ſun will be found, from the above data, to be 38° 19′.

Then to find the declination, the rule will be (according to the firſt caſe of oblique-angled ſpherical triangles) As the ſine of the hour from noon, 52° 30′; to the ſine of the azimuth, 72° 13′; ſo is the ſine of the complement of the altitude 38° : 19′, *viz.* 51° 41′; to the ſine of the complement of the declination 70° 21′, whence the declination will be 19° : 39′. 2d *Hodgſon*, 371, 155.

M *The*

The Practice on the QUADRANT.

Take 52° 30′ from the sines, in the compasses, with that extent enter one foot at the sine of 72° 13′, and bring the thread to the other; then take the sine of 51° 41′, in the compasses, from the centre of the sines, and entering, with that extent, between the thread and the scale of sines, they will rest at 70° 21′, the complement of 19° 39′, the declination required.

N. B. Since the altitude of the sun may be obtained by the Quadrant without calculation, this problem may, by that means, be answered by the last proportion alone, without the process in problem the 10th, above quoted.

PROBLEM III.

Prob. 3. *Given the altitude of the sun, his azimuth, and the time of the day; to find the declination.*

Here the operation is exactly the same, with that used to find the declination in the last problem; the altitude here being *given*, there being *found*.

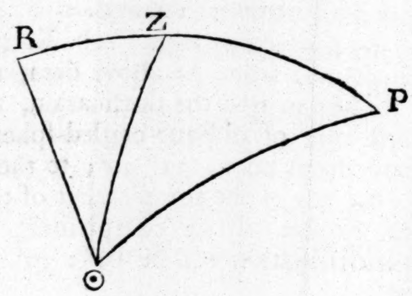
N. B. If the day were given, as well as the hour, the declination would appear by inspection on the Quadrant.

PROBLEM IV.

Prob. 4. *Given the latitude of the place, the altitude of the sun, and his azimuth; to find his declination.*

EXAMPLE.

In the latitude 51° 32′ north, the sun having 38° 19′ altitude, his azimuth at that time was found to be south 72° 13′ east, his declination is required.



In the oblique angled spherical triangle P Z ⊙, are given Z ⊙, the complement of the altitude; Z P, the complement of the latitude; and the angle at Z the azimuth; whence to find P ⊙, the complement of the declination.

Having let fall the perpendicular ⊙ R, the proportion will

be

be (by the ninth caſe of oblique-angled ſpherical triangles) As radius, to the co-ſine of the azimuth, 72° 13′, ſo is the co-tangent of the altitude, 38° 19′, to the tangent of a fourth arc ZR, 21° 08′. 2d *Hodgſon*, 367, 189, 193.

The Practice on the QUADRANT.

Changing the places of the middle terms, and then making the changed middle term to precede; the proportion will ſtand thus; As the tangent of the altitude, 38° 19′, to the radius, ſo is the co-ſine of the azimuth, 72° 13′, to the tangent of the fourth arc, 21° 08′: Take therefore, 38° 19′, from the tangents, in the compaſſes, with that extent enter one foot at radius on the tangents, and bring the thread to the other; then take 17° 47′, the complement of the azimuth, 72° 13′, from the centre of the ſines in the compaſſes, and entring, with that extent, between the thread and the ſcale of tangents, they will reſt at 21° 08′, the fourth arc ZR. *Collins* 73.

But $ZR + ZP = PR$, that is, if to the fourth arc, 21° 08′, be added the complement of the latitude, 38° 28′, the ſum is the arc $PR = 59° 36′$, a fifth arc.

21	08
38	28
59	36

And now (by the ſecond example of the tenth caſe of oblique angled ſpherical triangles,) to find the declination, the proportion will be, As the co-ſine of the fourth arc, 21° 08′, to the co-ſine of the fifth arc, 59° 36′, ſo is the ſine of the altitude, 38° 19′, to the ſine of the declination, 19° 39′. 2d *Hodgſon*, 368, 197.

The Practice on the QUADRANT.

Take 30° 24′, the complement of the fifth arc, from the ſines, in the compaſſes; with that extent enter one foot at 68° 52′, the complement of the fourth arc, on the ſines, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 38° 19′, the altitude, on the ſines, and with the other take the neareſt diſtance to the thread; this extent, applied to the centre of the ſines, will reach to 19° 39′, the declination required.

PROBLEM V.

Given the greateſt declination, and the right aſcenſion; to find the preſent declination. Prob. 5.

EXAMPLE.

The greateſt declination of the ſun, 23° 29′, and the right aſcenſion, 55° 17′, being given, the preſent declination is required. 2d *Hodgſon*, 253, 91.

The proportion will be (according to the ſecond example of the eighth caſe of right angled ſpherical triangles) As radius, to the ſine of the ſun's right aſcenſion, 55° 17′, ſo is the tangent of the greateſt declination, 23° 29′, to the tangent of the preſent declination, 19° 39′.

The Practice on the QUADRANT.

This problem is ſolved in the common way, without any difficulty; but is much ſooner done by placing the thread at 55° 17, on the line of ſines, at the limb of the Quadrant, for then it will cut the circle of declination at 19° 39′, the declination ſought.

PROBLEM VI.

Prob. 6. *Given the ſun's longitude and greateſt declination, to find his preſent declination.*

EXAMPLE.

2d *Hodgſon*, 246, 248, 80. On the 7th of May, at noon, the ſun's place in Taurus, 27° 34′, and his greateſt declination, 23° 29′, being given, thence to find his preſent declination.

The proportion will be (by the ſecond example of caſe the fifth of right angled ſpherical triangles) As radius, to the ſine of the ſun's longitude from Aries, 57° 34′, ſo is the ſine of his greateſt declination, 23° 29′, to the ſine of his preſent declination, 19° 39′.

27 34
30
57 34

This problem may be ſolved by the Quadrant, in the ſame manner as other proportions, but much ſooner thus, Lay the thread on ♉, 27° 34′, in the circle of ſigns, and it will cut the circle of declination at 19° 39′, the declination required.

PROBLEM VII.

Prob. 7. *Given the ſun's amplitude, and time of riſing; to find his declination.*

EXAMPLE.

Let the ſun's amplitude be 33° 38′, and the time of his riſing be at four hours ten minutes, which, converted into degrees, gives 62° 30′: Then ſay, As the ſine of 62° 30′, the angle from noon, to the ſine of 56° 22′, the complement of the amplitude, ſo is the radius, to the ſine of 69° 50′, the complement of the declination.

Patridge, 113.

The

The Practice on the QUADRANT.

Take 56° 22′ from the ſines, in the compaſſes; with that extent, enter one foot at 62° 30′, on the ſines, and bring the thread to the other foot; then enter one foot of the compaſſes (diſcharged of their firſt extent) at radius, and, with the other, take the neareſt diſtance to the thread; this extent, applied to the centre of the ſines, will reach to 69° 50′, the complement of the declination 20° 10′.

This problem may be alſo much eaſier performed by the Quadrant; for, if you lay the thread on the given time of the ſun's riſing, *viz.* four hours ten minutes, on the ſtrait line of hours, you will find it cuts the tenth of May in the lower arc of months; and then laying the ſtring on the tenth of May, in the upper circle of months, it will cut the declination underneath at 20° 10′.

PROBLEM VIII.

Prob. 8. 1ſt *Leadbetter*, 165.

Given the latitude of the place, and the hour of the ſun's ſetting; to find his declination.

EXAMPLE.

At London, when the ſun apparently riſes at five and ſets at ſeven, his declination is required.

Here the time five hours from midnight, reduced into degrees, is 75°, its complement 15°; the latitude is 51° 32′; its complement 38° 28′; and the proportion will be, As the ſine of 15°, the complement of the hour, to radius, ſo is the tangent of 51° 32′, the latitude, to the tangent of 78° 23′, the complement of the declination.

The Practice on the QUADRANT.

Change the two firſt terms, which will infer a change of the tangents into co-tangents, and then it will be, As radius, to the ſine of 15°, ſo is the tangent of 38° 28′, to the tangent of 11° 37′, the declination.

Take 15° from the ſines, in the compaſſes; with that extent, enter one foot at radius, on the tangents, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at the tangent of 38° 28′, and take the neareſt diſtance to the thread; this extent, applied to the centre of the tangents, will reach to 11° 37′, the declination.

But

But the Quadrant gives here alſo, a much eaſier method of ſolving this problem, thus: Lay the thread at the hour of five on the ſtrait line of hours, and the day cut thereby, in the circle of months underneath, is the 10th of April; then lay the thread to the 10th of April on the upper circles of months, and it cuts the circle of declination in 11° 37′, as above.

PROBLEM IX.

Prob. 9. *Given the latitude of the place, and the ſun's amplitude; to find his declination.*

EXAMPLE.

1ſt *Leadbetter*, 166. At London, in the latitude 51° 32′, when the ſun riſes and ſets 10° to the northward of the eaſt and weſt points; his declination is required.

In this caſe are given the amplitude, 10°, from the eaſt or weſt, and the latitude 51° 32′, its complement being 38° 28′; to find the declination. And the proportion will be (according to caſe the fifth of right angled ſpherical triangles,) As radius, to the ſine of 10° 00′, ſo is the ſine of 38° 28′, the complement of the latitude, to the ſine of the declination, 6° 12′ north.

The Practice on the QUADRANT.

Take 10° from the ſines, in the compaſſes; with that extent, enter one foot at radius, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of their firſt extent) at 38° 28′ on the ſines, and, with the other, take the neareſt diſtance to the thread; this extent, applied to the centre of the ſines, will reach to 6° 12′, the declination required, which is north, becauſe the amplitude is ſo.

HOURS.

PROBLEM I.

The sun being in the prime vertical, there is given his altitude, and present declination; to find the hour. Prob. 1.

EXAMPLE.

The sun having 19° 39′ north declination, and 25° 26′ of altitude, when upon the prime vertical; the hour of the day is required. 2d *Hodgson*, 306, 110.

The proportion will be (by the 14th case of right angled spherical triangles) As the sine of 70° 21′, the complement of the declination, 19° 39′, to the radius, so is the sine of 64° 34′, the complement of the altitude, 25° 26′, to the sine of the hour from noon, 73° 32′.

The Practice on the QUADRANT.

Take 70° 21′ from the sines, in the compasses, with that extent, enter one foot at radius, and bring the thread to the other; then take 64° 34′ from the centre of the sines in the compasses, and with that extent, entering between the thread and the scale of sines, they will rest at 73° 32′, which, reduced to time, is equal to 4 hours 54 minutes, at which time the sun will appear due west in the afternoon; and this, taken from 12 hours, will leave 7 hours 6 minutes, for the time when he will appear due east in the morning.

PROBLEM II.

The sun being in the equator, there is given the latitude of the place, and the altitude of the sun; to find the hour of the day. Prob. 2.

EXAMPLE.

In the latitude 51° 32′, the sun being in the equator, his altitude is found to be 22° 15′; the hour of the day is required. 2d *Hodgson*, 315, 97.

In this case are given 38° 28′, the complement of the latitude, 51° 32′, and the sun's altitude, 22° 15′; and the proportion will be

be (by case the 10th of right angled spherical triangles,) As the cosine of the latitude, 38° 28′, to the radius, so is the sine of the altitude, 22° 15′, to the sine of 37° 30′, the complement of the hour from noon, which, therefore, is 52° 30′.

The Practice on the QUADRANT.

Take 38° 28′ from the sines, in the compasses; with that extent, enter one foot at radius on the sines, and bring the thread to the other; then take 22° 15′ from the centre of the sines, in the compasses, and with that extent, entering between the thread and the scale of sines, they will rest at 37° 30′, the complement of 52° 30′, the hour from noon, which being reduced to time, will give 3 hours and 30 minutes, for the time in the afternoon; and that, subtracted from 12 hours, leaves 8 hours 30 minutes for the time in the morning.

PROBLEM III.

Prob. 3. *Given the azimuth, and altitude, the sun being in the equator; thence to find the hour of the day.*

EXAMPLE.

2d Hodgson, 317, 66. The sun being in the equator, his altitude was found to be 22° 15′, and at the same time his azimuth from the south was 59° 00′; the hour of the day is required.

In this case the proportion will be (according to case the first of right angled spherical triangles) As radius, to the sine of the azimuth from noon, 59° 00′, so is the sine of 67° 45′, the complement of the altitude, to the sine of the hour from noon, 52° 30′.

The Practice on the QUADRANT.

Take 59° 00′ from the sines, in the compasses; with that extent, enter one foot at radius, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at 67° 45′, on the sines, and with the other take the nearest distance to the thread; this extent, applied to the centre of the sines, will reach to 52° 30′, the hour from noon, which being reduced to time, becomes 3 hours 30 minutes, the time in the afternoon; and this, subtracted from 12 hours will leave 8 hours 30 minutes for the hour of the day in the forenoon.

PROBLEM IV.

Given the latitude, 51° 32′; *declination,* 23° 29′; *and altitude of the sun,* 36° 38′; *to find the hour of the day, according to Mr. Collins's method, page* 182. Prob. 4.

Now to the declination — — — — — 23° : 29′
Add the complement of the latitude, 51° 32′, *viz.* 38° : 28′

The sum will be the sun's meridian altitude — —61° : 57′

The canon is, As the co-sine of the declination, 23 29′= 66° 31′, to the secant of the latitude; or, As the co-sine of the latitude, to the secant of the declination, so is the difference of the sines of the sun's observed and meridian altitude, to the versed sine of the hour from noon. *Collins,* 182.

Operation of the first Proportion, viz.

As the co-sine of the declination, to the secant of the latitude, *&c.*

Take the distance between the observed and meridian altitudes, *viz.* 61° 57′, and 36° 38′, on the line of sines, and enter it twice down the same line of sines, from the centre, and let one foot of the compasses rest there; lay the thread over the secant of the latitude, 51° 32′, in the arc of secants, and extend or contract the compasses, without removing the foot from the place where it rests, till the other foot touches the thread; with the extent, thus obtained, enter one foot again at the complement of the declination, 66° 31′, on the sines, and bring the thread to the other foot; then will the thread cut the circular versed sines at 60°, which, turned into time, gives four hours from noon, whether you count eight in the morning, or four in the afternoon.

Note here, that the circle of hours, underneath the circular versed sines, is fitted to that line, and eases the trouble of converting the sines into time.

The same kind of operation serves for the other proportion *mutatis mutandis,* and you will find, that if the thread is laid, as above, over the secant of 51° 32′, the extent must be entred at the sine of 66° 31′, but if it is laid at 23° 29′, it must be entred at 38° 28′.

Note also, That in the rule given by Mr. Collins, the entrance is to be only once drawn down the sines, but (as above) it is here directed to be entred twice; the reason of which is, Mr. Collins works by the line of versed sines of 180°, whereas, on Mr. Rowley's Quadrant, the versed sines are continued only to 90°.

PROBLEM V.

Prob. 5. *The sun's altitude, declination, and azimuth, being given; to find the hour of the day.*

Example.

2d Hodgson, 337. Collins, 163. The sun having 19° 39′ north declination, his altitude was found to be 38° 19′, and, at the same time, his azimuth was south 72° 13′ east; the hour of the day is required.

2d Hodgson, 155, 156. The proportion will be (according to the first case of oblique angled spherical triangles) As the sine of 70° 21′, the complement of the declination, 19° 39′, to the sine of the azimuth, 72° 13′, so is the sine of 51° 41′, the complement of the sun's altitude, 38° 19′, to the sine of the hour from noon, 52° 30′, which, converted into time, gives three hours thirty minutes, if the altitude was taken in the afternoon; and eight hours thirty minutes, if taken in the morning.

The Practice on the Quadrant.

Take 70° 21′ from the sines, in the compasses; with that extent, enter one foot at 72° 13′, and bring the thread to the other; then take 51° 41′ from the centre of the sines, in the compasses, and entring, with that extent, between the thread and the scale, they will rest at 52° 30′.

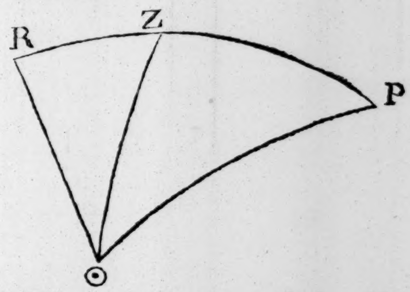
PROBLEM VI.

Prob. 6. *Given the latitude of the place, the azimuth of the sun, and his altitude; to find the hour of the day.*

Example.

2d Hodgson, 367, 193. In the latitude of 51° 32′ north, the sun having 38° 19′ of altitude, his azimuth was found to be south 72° 13′ easterly; the hour of the day is required.

This



This problem requires the solution of two proportions, and has the same things given as in Prob. IV. of finding the sun's declination, page 82; therefore the fourth arc Z R = 21° 8′, and the fifth arc P R = 59° 36′, may be found in the same manner as in that Problem.

Then the remaining proportion will be, As the sine of the fifth arc, 59° 36′, to the sine of the fourth arc, 21° 08′, so is the tangent of the azimuth, 72° 13′, to the tangent of the hour from noon, 52° 30′. It will hold also, As the sine of the fourth arc, 21° 08′, to the sine of the fifth arc, 59° 36′, so is the co-tangent of (72° 13′ =) 17° 47′, to the co-tangent of (52° 30′ =) 37° 30′. 2d *Hodgson*, 368.

The Practice on the QUADRANT.

Change the two middle terms, to avoid bringing the point of the compasses from the tangents to the sines; and then it will be, As the sine of 21° 08′, to the tangent of 17° 47′, so is the sine of 59° 36′, to the tangent of 37° 30′.

Now take 17° 47′ from the tangents, in the compasses; with that extent, enter one foot at the sine of 21° 08′, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at 59° 36′ on the sines, and thence take the nearest distance to the thread; this extent, applied to the centre of the tangents, will reach to 37° 30′, the complement of 52° 30′, the hour from noon; and this, being converted into time, gives three hours thirty minutes; which (as the azimuth was easterly) being deducted from twelve hours, leaves eight hours thirty minutes for the hour before noon.

LATITUDE.

PROBLEM I.

Prob. 1. *Given the azimuth of the sun, his declination, and hour of the day; to find the latitude of the place.*

EXAMPLE.

2d Hodgson. 355, 178. THE sun having 19° 39′ north declination, his azimuth, at eight hours thirty minutes in the morning, was found to be south 72° 13′ east; the latitude is required.

In this example are given the complement of the declination (19° 39′ =) 70° 21′, the complement of the hour from noon (52 30′ =) 37° 30′; the complement of the azimuth (72° 13′ =) 17° 47′; thence to find the complement of the latitude; and the proportion will be, according to the sixth case of oblique spherical triangles (the place of the two first terms being changed) As the co-sine of the hour from noon, 37° 30′, to the radius, so is the tangent of the declination, 19° 39′, to 30° 24′, the co tangent of a fourth arc, which, therefore, is 59° 36′.

And, again, it will be, As the co-tangent of the hour from noon, 37° 30′, to the co-tangent of the azimuth, 17° 47′, so is the sine of the fourth arc, 59° 36′, to the sine of a fifth arc, 21° 08′. Now, if from the fourth arc, 59° 36′, be taken the fifth arc, 21° 08′, there will remain the co-latitude, 38° 28′; and this, taken from 90°, gives the latitude 51° 32′.

Or, if to the complement of the fourth arc, 30° 24′, be added the fifth arc, 21° 08′, the sum will be the latitude, 51° 32′.

The Practice on the QUADRANT, *in the first proportion*, viz.

As the co-sine of the hour from noon, 37° 30′, to the radius, so is the tangent of the declination, 19° 39′, to 30° 24′, the co-tangent of the fourth arc, 59° 36′.

Take 37° 30′ from the sines, in the compasses; with that extent, enter one foot at radius on the tangents, and bring the thread to the other;

3

other; then take 19° 39′ from the centre of the tangents, in the compaſſes, and, with that extent, entring between the thread and the ſcale of tangents, they will reſt at 30° 24′, the complement of 59° 36′.

Again, in the ſecond proportion. Take 17° 47′, the complement of the azimuth, 72° 13′, from the centre of the tangents, in the compaſſes; with that extent, enter one foot in the ſcale of ſines, at the ſine juſt oppoſite to the tangent of 37° 30′, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at the ſine of 59° 36′, and with the other take the neareſt diſtance to the thread; this extent, ſet to the centre of the ſines, will reach to the ſine of 21° 08′, the fifth arc, as above.

Obſerve, That if the extent of 17° 47′, on the tangents, is applied to the tangent of 37° 30′, and the ſtring is carried back 3° 20′ on the limb of the Quadrant, and then the neareſt diſtance is taken between the tangent of 37° 30′, and the ſtring ſo carried back 3° 20′, it will then produce the ſame 21° 08′, as is above found, of which a caution or notice is given in the firſt part of this treatiſe.

PROBLEM II.

Given the ſun's declination, and his altitude, at the hour of ſix; thence to find the latitude. Prob. 2.

EXAMPLE.

The ſun having 19° 39′ north declination, and his altitude, at the hour of ſix, being found to be 15° 16′; thence to find the altitude. 2d *Hodgſon*, 297, 298, 107.

The proportion will be (by the thirteenth caſe of right angled ſpherical triangles) As the ſine of the ſun's declination, 19° 39′, to the radius, ſo is the ſine of his height at the hour of ſix, 15° 16′, to the ſine of the latitude, 51° 32′, which is north becauſe the declination is north.

This problem may be worked on the Quadrant in the manner already ſufficiently explained.

PRO-

PROBLEM III.

Prob. 3. *Given the sun's declination, and azimuth at six; to find the latitude.*

Example.

2d *Hodgson*, 298, 299, 105, 106. The sun having 19° 39′ north declination, his azimuth at six in the morning, was found to be 77° 28′ east; thence to find the latitude.

The proportion will be (by the twelfth case of right angled spherical triangles, making radius the second term) As the tangent of the declination, 19° 39′, to the radius, so is the co-tangent of the azimuth, 77° 28′, to the sine of 38° 28′, the complement of the latitude, 51° 32′.

The Practice on the Quadrant.

Take 19° 39′ from the tangents, in the compasses; with that extent, set one foot at radius on the sines, and bring the thread to the other; then take 12° 32′, the complement of 77° 28′, the azimuth from the centre of the tangents, and, with this extent, entring the compasses between the thread and the scale of sines, they will rest on 38° 28′, the complement of the latitude, 51° 32′, which is north, because the declination is north.

PROBLEM IV.

Prob. 4. *The sun being on the prime vertical, the altitude and declination are given; to find the latitude.*

Example.

The sun having 19° 39′ north declination, and 25° 26′ of altitude, and being on the prime vertical; the latitude is required.

2d *Hodgson*, 106. To find which, the proportion will be (by the thirteenth case of right angled spherical triangles) As the sine of the altitude, 25° 26′, to the sine of the declination, 19° 39′, so is radius, to the sine of the latitude, 51° 32′, which is north, because the declination is north.

The practice of this proportion on the Quadrant, has been already sufficiently exemplified.

PRO-

PROBLEM V.

The sun being on the prime vertical, there is given the declination, and the time of the day; to find the latitude. Prob. 5.

EXAMPLE.

The ſun having 19° 39′ north declination, was obſerved to be upon the prime vertical, at four hours fifty-four minutes, afternoon; the latitude is demanded.

In this example four hours fifty-four minutes is equal to 73° 30′; and the proportion will be (according to the ſecond caſe of right angled ſpherical triangles,) As the tangent of the declination, 19° 39′, to radius, ſo is the ſine of 16° 30′, the complement of the hour from noon, to the tangent of 38° 28′, the complement of the latitude, which, therefore, is 51° 32′. 2d *Hodgſon*, 69.

The Practice on the QUADRANT.

Take 19° 39′ from the tangents, in the compaſſes; with that extent, enter one foot at radius on the tangents, and bring the thread to the other; then take 16° 29′, in the compaſſes, from the ſines, and, with that extent, entring between the thread and the ſcale of tangents, it will reſt at 38° 28′, the complement of the latitude 51° 32′.

PROBLEM VI.

The sun being in the equator, there is given his altitude, and azimuth; to find the latitude. Prob. 6.

EXAMPLE.

The ſun being in the equator, his altitude was found to be 22° 15′, and, at the ſame time, his azimuth from the ſouth was 59° 00′ eaſterly; the latitude is required.

And the proportion will be (by caſe the ſecond of right angled ſpherical triangles,) As the ſine of 31° 00′, the complement of the azimuth from the meridian, to the radius, ſo is the tangent of the altitude, 22° 15′, to the tangent of 38° 28′, the complement of the latitude, 51° 32′. 2d *Hodgſon*, 67, 68, 3 8.

This

3

This problem may be resolved by the Quadrant in the usual manner, observing only, to apply the sine of 31° 00′, to radius on the tangents, as the last proportional is to be taken there.

PROBLEM VII.

Prob. 7. *The sun being in the equator, there is given his altitude, and the hour of the day; to find the latitude.*

EXAMPLE.

The sun being in the equator at thirty minutes after eight in the morning (which, turned into degrees, is 52° 30′) his altitude was found to be 22° 15′; the latitude is required.

2d Hodgson, 67, 320. The proportion will be (by the second case of right angled spherical triangles,) As the sine of 37° 30′, the complement of the hour from noon, 52° 30′, to the radius, so is the sine of the altitude, 22° 15′, to the sine of 38° 28′, the complement of the latitude, 51° 32′.

This problem, likewise, may be solved on the Quadrant, in the common way, without any difficulty.

PROBLEM VIII.

Prob. 8. *The sun having 19° 39′ north declination, his altitude was observed to be 38° 19′, and, at the same time, his azimuth was south 72° 13′ east; the latitude is required.*

2d Hodgson, 338, 160. The proportion will be (according to case the third of oblique angled spherical triangles,) As the radius, to the sine of 17° 47′, the complement of the azimuth, 72° 13′, so is the tangent of 51° 41′, the complement of the altitude, 38° 19′, to the tangent of a fourth arc, 21° 08′.

The Practice on the QUADRANT.

Change the two middle terms, and then take 38° 19′ from the tangents, in the compasses; with that extent, enter one foot at radius on the tangents, and bring the thread to the other; then take 17° 47′, from the sines, in the compasses, and entring, with that extent, between the thread and the scale of tangents, they will rest at the fourth arc, 21° 08′.

Again,

Again, the next proportion will be, As the sine of the altitude, 38° 19′, to the sine of the declination, 19° 39′, so is the sine of 68° 52′, the complement of the fourth arc, 21° 8′, to the sine of 30° 24′, the complement of a fifth arc, 59° 36′.

The Practice on the QUADRANT.

Take the sine of 19° 39′ in the compasses; with this extent, enter one foot at 38° 19′ on the sines, and bring the thread to the other; then enter one foot of the compasses (discharged of the first extent) at 68° 52′ on the sines, and, with the other, take the nearest distance to the thread; this extent, applied to the centre of the sines, will reach to 30° 24′, the complement of the fifth arc, 59° 36′.

If from the fifth arc, 59° 36′, be taken the fourth arc, 21° 8′, there will remain the complement of the latitude, 38° 28′, and from 90° take the above, 38° 28′, and there remains for the latitude 51° 32′.

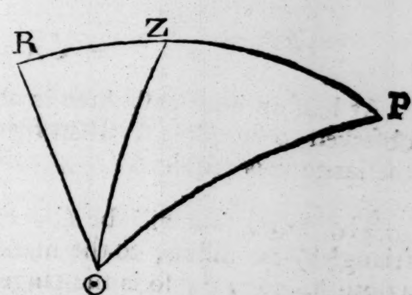
PROBLEM IX.

Given the altitude of the sun, and his present declination, with the hour of the day; to find the latitude. Prob. 9.

EXAMPLE.

The sun having 19° 39′ north declination, at three hours thirty minutes in the afternoon, equal in degrees to 52° 30′, his altitude was found to be 38° 19′; the latitude is required.

This case is doubtful, that is, the same things may happen in two different latitudes.



2d *Hodgson*, 342, 160.

In order to determine these, let fall the perpendicular ⊙ R, and the proportion will be (by the third case of oblique spherical triangles) As radius, to the sine of 37° 30′, the complement of the hour from noon, 52° 30′, so is the tangent of 70° 21′, the complement of the declination, 19° 39′, to the tangent of a fourth arc, 59° 36′; but, because the tangents of 70° 21′, and of 59° 36′ exceed the limits of the Quadrant, let the second term precede the first, which will infer a change of the tangents into co-tangents, and the analogy will then be, As the co-sine of the hour from noon, 37° 30′, to the radius, so is the tangent

O of

of the declination, 19° 39′; to the tangent of 30°: 24′ the complement of 59° 36′ the fourth arc.

The Practice on the QUADRANT.

Take 37° 30′ from the ſines in the compaſſes; with that extent enter one foot at radius on the tangents, and bring the thread to the other; then take 19° 39′ from the tangents, in the compaſſes, and with that extent, entring between the thread and the ſcale of tangents, they will reſt at 30° 24′ the complement of 59° 36′ the fourth arc.

Again, the ſecond proportion will be, as the ſine of the declination, 19° 39′; to the ſine of the altitude, 38° 19′; ſo is the ſine of 30° 24′, the complement of the fourth arc, 59° 36′; to the ſine of 68° 52′ the complement of a fifth arc 21° 8′.

This may be worked on the Quadrant in the common way: Then, if to the fourth arc, 59° 36′ be added the fifth arc, 21° 8′; the ſum will be 80° 44′, which taken from 90°, leaves 9° 16′ for the leſſer latitude; but if from the fourth arc, 59° 36′, be taken the fifth arc 21° 8′; there remains 38° 28′, and this taken from 90°, gives 51° 32′, for the greater latitude.

PROBLEM X.

Prob. 10. *Given the altitude of the ſun, his azimuth, and time of the day, to find the latitude.*

EXAMPLE.

2d *Hodgſon*, 358. At half an hour paſt three in the afternoon, the ſun's altitude was obſerved to be 38° 19′, when his azimuth was ſouth 72° 13′ weſt; the latitude is required.

2d *Hodgſon*, 178. The proportion will be (by caſe the ſixth of oblique ſpherical triangles) As radius, to the ſine of 17° 47′, the complement of the azimuth, 72° 13′; ſo is the tangent of 51° 41′, the complement of the altitude, 38° 19′; to the tangent of a fourth arc 21° 8′.

The Practice on the QUADRANT.

Here, becaufe the tangent of 51° 41′ exceeds the limits of the Quadrant, let this (being a middle term) be changed into its complement, 38° 19′, and be made the firft term; and then the analogy will be, As the tangent of 38° 19′, to radius; fo is the fine of 17° 47′, to the tangent of 21° 08′.

Take 38° 19′ from the tangents, in the compaffes; with that extent, enter one foot at radius on the tangents, and bring the thread to the other; then take 17° 47′ from the centre of the fines, and entering that extent between the thread and the tangents, it will reft at 21° 8′, the fourth arc.

Again, the fecond proportion will be, As the tangent of the hour from noon, 52° 30′, is to the tangent of the azimuth, 72° 13′; fo is the fine of the fourth arc, 21° 8′, to the fine of a fifth arc, 59° 36′.

The Practice on the QUADRANT.

Let the two firft terms change places, and their tangents will be changed into co tangents; and then it will be, As the co-tangent of (72° 13′ =) 17° 47′, to the co-tangent of (52° 30′ =) 37° 30′, fo is the fine of 21° 8′, to the fine of a fifth arc, 59° 36′.

Take 17° 48′ from the tangents, in the compaffes; with that extent, enter one foot at the fine oppofite to the tangent of 37° 30′, and bring the thread to the other; then take 21° 8′ from the fines, in the compaffes, and with that extent, entring between the thread and the fcale of fines, they will reft at 59° 36′, the fifth arc.

Or, it will hold thus, As the tangent of 17° 47′, to the fine of 21° 8′, fo is the co-tangent of (52° 30′ =) 37° 30′, to the fine of the fifth arc, 59° 36′, and, working in this way, the compafs point need not be brought down from the tangents to the oppofite fines; nor need it fo to be, if the thread is removed back 3° 20′, as before obferved.

Now the fifth arc, 59° 36′, leffened by the fourth arc, 21° 8′, will give 38° 28′, for the co-latitude of the place; and this, taken from 90°, leaves 51° 32′, for the latitude.

PROBLEM XI.

Prob. 11. *Given the ſun's amplitude, and the declination; to find the latitude.*

EXAMPLE.

2d *Hodgſon*, 290. The amplitude of the ſun being 32° 44′ northerly in the morning, and the declination being north 19° 39′; thence to find the latitude.

2d *Hodgſon*, 107. The proportion will be (by the thirteenth caſe of right angled ſpherical triangles,) As the ſine of the amplitude, 32° 44′, to the ſine of the declination, 19° 39′, ſo is the radius, to the co-ſine of the latitude, (51° 32′ =) 38° 28′, which is north, if the ſun riſes before, or ſets after ſix.

This will be performed by the Quadrant in the common Form.

PROBLEM XII.

Prob. 12. *The meridian altitude of the ſun, and the declination given; thence to find the latitude.*

EXAMPLE.

Suppoſe the meridian altitude, found by the Quadrant, or otherwiſe, to be 58° 28′, and the declination 20° north, then the complement of the meridian altitude, which is always equal to the ſun's meridional zenith diſtance, will in this caſe be 31° 32′, which being added to 20° 00′, the declination, gives 51° 32′, the latitude required.

LONGI-

Longitude *of the* SUN, *and his* Place.

PROBLEM I.

The latitude of the place, and the declination of the ſun being given; thence to find the time when he will be upon the prime vertical, or due eaſt and weſt. Prob. 1.

Example.

In the latitude 51° 32′ north, the ſun's declination being 19° 39′ north; the time when he will be upon the prime vertical is required.

The proportion will be (by the ninth caſe of right angled ſpherical triangles) As the tangent of the latitude, 51° 32′, to the radius, ſo is the tangent of the declination, 19° 39′, to the ſine of the time required, 16° 28′. Or, in order to bring the tangent of the latitude, 51° 32′, within the limits of the Quadrant, it will hold, As radius, to the tangent of 38° 28′, the complement of the latitude, 51° 32′, ſo is the tangent of the declination, 19° 39′, to the ſine of the time required, in degrees 16° 28′. 2d *Hodgſon*, 303, 95.

The Practice on the Quadrant.

Take 38° 28′ from the tangents, in the compaſſes; with that extent, enter one foot at radius, on the tangents, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 19° 39′ on the tangents, and, with the other, take the neareſt diſtance to the thread; this extent, applied to the centre of the ſines, will reach to 16° 28′, the ſine of the time required; which, reduced to time, will be one hour ſix minutes, before or after ſix.

And note, that by placing the foot of the compaſſes at radius, on the tangents, the 3° 20′, taken notice of before, are regained, as in ſeveral other beforegoing Inſtances.

PRO-

PROBLEM II.

Prob. 2. *Given the latitude of the place, and the sun's altitude at the true east and west points; to find the hour and minute when he will be there.*

EXAMPLE.

Gunter, 266. Given the latitude, 51° 32′, and the sun's height, found by observation, or otherwise, 25° 26′; thence to determine the Question.

Hawney, 360. The proportion will be, As radius, to the sine of 38° 28′, the complement of the latitude, 51° 32′, so is the tangent of the sun's altitude, 25° 26′, to the tangent of the hour from six, 16° 28′.

The Practice on the QUADRANT.

Take 38° 28′ from the sines, in the compasses; with that extent, enter one foot at radius, on the tangents, and bring the thread to the other; then enter one foot of the compasses (discharged of their first extent) at the tangent of 25° 26′, and with the other take the nearest distance to the thread; this extent, applied to the centre of the tangents, will reach to 16° 28′ = 1 hour 6 minutes, before or after six.

PROBLEM III.

Prob. 3. *Given the greatest declination,* 23° 29′, *and the sun's right ascension,* 55° 17′; *thence to find the sun's place or longitude.*

The proportion will be, As radius, to the sine of 66° 31′, the complement of the greatest declination, 23° 29′, so is the tangent of 34° 43′, the complement of the right ascension, 55° 17′, to the tangent of 32° 26′, the complement of the sun's longitude, 57° 34′.

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The Practice on the QUADRANT.

Take 66° 31′ from the ſines, in the compaſſes, with that extent enter one foot at radius on the tangents, and bring the thread to the other; then enter one foot of the compaſſes (diſcharged of the firſt extent) at 34° 43′ on the tangents, and, with the other, take the neareſt diſtance to the thread; this extent, applied to the centre of the tangents, gives 32° 26′, the complement of the correſponding longitude, 57° 34′.

The Practice on the QUADRANT, *in a much ſhorter way.*

Set the thread to the given right aſcenſion, 55° 17′, on the limb of the Quadrant, and the thread, thus laid, will cut his correſponding longitude at ♉ (27° 34′=) 57° 34′, on the ecliptic, counting from ♈.

PROBLEM IV.

Given the ſun's greateſt and preſent declination; to find his longitude. Prob. 4.

EXAMPLE.

The ſun's greateſt declination is 23° 29′, his preſent declination 19° 39′, north; thence to find his longitude. 2d *Hodgſon*, 250, 97.

The proportion will be (by the 10th caſe of right angled ſpherical triangles) As the ſine of the ſun's greateſt declination, 23° 29′, to the ſine of the ſun's preſent declination, 19° 39′, ſo is the radius, to the ſine of the preſent longitude, 57° 34′.

This may be practiſed on the Quadrant, in the common way.

STARS.

STARS.

Observations *necessary to precede the following* Cases.

Obs. I. *Collins* 25. I. In finding the time of the night by the stars, no more than twelve hours of right ascension are made use of, for their rising or setting.

Obs. II. *Hodgson's* first System, 620. II. Because the right ascension, declination, meridian altitude, or zenith distance, oblique ascension and descension, ascensional difference, and amplitude of a star, may be found in the same manner as those of the sun; the methods therefore of treating these, and resolving the questions depending on them, are reciprocally the same; and this may serve to shorten the work relating to the stars in these respects.

Obs. III. III. In the following discourse, and others in the printed books, you will meet with the terms, the *sun's hour*, and *star's hour*; to define and explain which, observe, that as the celestial bodies appear to make an intire revolution in the space of twenty-four hours, consequently the twenty fourth part of the circumference of the equinoctial line, consisting of 360°, or, which is the same thing, 15 degrees, will transit or pass over the meridian in an hour. And hence it is, that the arc of the equinoctial, intercepted between the right ascension of the sun, and any fixed star, reduced to time, shews how long that star will transit the meridian after the sun, and, consequently, the time it will culminate.

Suppose, for instance, the sun's right ascension was 300 degrees, and that of a star 345 degrees, the difference being 45 degrees; this, converted into time, is three hours, wherefore, the sun's hour of right ascension is three hours before that of the star's; the first, therefore, is the *sun's hour*, the other the *star's hour*; and, in counting on the *Globe*, the intervening degrees upon the equinoctial between the sun's and star's right ascension, you are not to stop at 360 degrees, if the case happens to require more, but go over to those that follow. As, suppose the sun's right ascension (found on the Quadrant as before) was 300 degrees, and the star's 20 degrees, the difference will then be 80 degrees, *viz.* 60° to compleat the 300° to 360°, and 20 more from the equinoctial point.

The right ascension of a star may be found on the celestial globe thus; bring the star to the meridian, and the brazen circle will cut the equinoctial in the right ascension, from the first point of Aries; or, the right ascension may be found by taking the hour of such star's right ascension, as noted against it on the Quadrant, and adding 12 thereto, where the mark + is affixed; this hour, converted into degrees, by multiplying the same by 15°, will give the right ascension required.

For instance, against Arcturus, marked with a +, in the lowest circle of ascensions, is 2 hours; add to it 12, and the sum 14 being multiplied by 15, gives the right ascension, 210°.

From the consideration of the right ascension of the stars, let us proceed to find the time of their Culmination, or *Southing*; for which several ways are prescribed.

The first is this, subtract the right ascension of the sun from that of the star, increased by 180, or 360 degrees, if necessary; and the remainder, converted into time, shews the star's southing.

Mr. Collins, in pages 24 and 34, applies this rule to the Quadrant; you must (says he) make use of the sun's whole right ascension, converted into time, as it is found on the Quadrant; as also the star's whole right ascension, to be taken from the circles of their right ascension on the Quadrant (of which more hereafter;) then subtract the sun's whole right ascension from that of the star's, increased by 12 hours, if the star has the mark + affixed to it; but if 12 hours are not sufficient for subtraction, then, instead of 12, make it 24 hours, and the remainder, if less than 12, shews the time in the afternoon or night, when the star will be upon the meridian; but, if there remains more than 12 hours, reject 12, and the residue is the time of the next morning when the star will be upon the meridian: And, in page 34, he adds, *generally*, that to get the difference between the two ascensions, you must subtract the less from the greater; *Collins* 34. this remainder is to be added to the star's hour when the star is before the sun; but otherwise, to be subtracted from it; or, as Mr. Hodgson (in his treatise of Navigation, page 372) more clearly expresses it, subtract the right ascension of the sun from that of the star, and the remainder shews the time of the star's coming to the meridian; but, if it happens that the sun and star are on contrary sides of the first point of Aries, that is, if the sun's right ascension exceeds that of the star's, then to the star's right ascension add 24 hours; after which make the subtraction.

And this makes way for the explication of thoſe lines on the Quadrant that are neceſſary to anſwer to the above rules: It has been taken notice of already, that underneath the rectifying table on the Quadrant, there are four quadrantal annuli, the loweſt of which contains fixed ſtars, oppoſite to each of which are contained, in the next annulus, their names; in the third are their declinations; and, in the fourth is marked whether the declination is north or ſouth.

Below theſe are two annuli called Quadrants of Aſcenſion, reckoned from the left to the right, the higheſt 6, 7, 8, *&c.* and the loweſt 1, 2, 3, *&c.* to 12; the thread being laid over any ſtar cuts the lower line at the hour of right aſcenſion, if that ſtar riſeth after the next preceding equinoctial point; and the higher line is cut by it at the hour of right aſcenſion, if the ſtar riſeth after the next preceding ſolſtitial point.

For inſtance, the thread laid over the bright ſtar of Aries, cuts the lower annulus at 1 hour 51 minutes, which, converted into degrees, is $27^{\circ}\ 56'$, reckoned from the preceding equinoctial point.

Again, the right aſcenſion of Ala Pegaſi upon the equinoctial line, is $342^{\circ}\ 46'$, which, converted into time, is 22 hours 48 minutes, and, rejecting 12 hours, is 10 hours 48 minutes; and ſo much the thread cuts over the lower annulus, reckoning the time from the equinoctial point Libra: But as this ſtar riſes after the preceding ſolſtice ♑, whoſe right aſcenſion is 18 hours, the ſame taken from the abovemention'd 22 hours 48 minutes, leaves 4 hours 48 minutes for the right aſcenſion of this ſtar in time, and, accordingly, the thread laid over the ſtar, cuts the upper or ſtolſtitial annulus at 4 hours 48 minutes, and if 6 hours are added to 4 hours 48 minutes, the ſum is 10 hours 48 minutes, which is equal to the time cut in the lower or equinoctial annulus. This difference of 6 hours, or 90 in degrees, the diſtance between the equinoctial and ſolſtitial points, runs through the two annuli, ſo that if you ſubtract the hours in the lower annulus, as far as the number 6, from the hours of the firſt half of the upper one, and, contrariwiſe, ſubtract the hours in the upper annulus from the lower, in the next halves of theſe annuli, the difference will be throughout 6 hours; and, therefore, it will be the ſame thing, as far as the numbers of the annuli extend, whether you work by the upper annulus for the ſtar's right aſcenſion from the next preceding ſolſtitial point, or by the lower annulus for the ſtar's right aſcenſion from the next preceding equinoctial point, making the proper allowance of 6 hours. But here obſerve, that as to all thoſe ſtars, which are marked in the Quadrant with this mark (+) ſignifying *plus*; to

theſe,

these, 12 hours must be added, as they exceed 180° in right ascension, which is equal to 12 hours.

However, that the reader may be at no trouble in seeking after the right ascension of the stars named in the Quadrant, so far as to degrees and minutes of a degree, or of converting those degrees and minutes of a degree into time, or of resorting to the celestial globe to find whether the right ascension of the stars referred to in the Quadrant, is after one or the other of the equinoctial or solstitial points; I have made or formed the annexed table, by which (as to any stars set on the Quadrant) he will see at once, after which of the equinoctial or solstitial points, the star sought rises; together with its right ascension, as well in degrees as time; and with these he will have (in a distinct column) the several declinations of those stars that are so taken notice of in the Quadrant.

A TABLE, *containing the right ascension of such stars as are marked on the* QUADRANT, *both in degrees and in time; shewing their rising after either of the equinoctial or solstitial points, and their several declinations.*

Names of the stars.	Right ascension in degrees and minutes after the equinoctial points.	Ditto in time, as *per* Quadrant.	Declinations according to the Tables and Quadrant *propè*.
+ Virgin's Spike	17° 42′ after Libra	1^h 11′	9° 43′ S.
Bright * Aries	27 56 after Aries	1 52	22 08 N.
+ Arcturus.	30 47 after Libra	2 03	20 38 N.
+ Bright * No Cer.	50 46 after Libra	3 23	27 39 N.
Bull's Eye	65 02 after Aries	4 20	15 55 N.
Orion's left Shoulder	77 37 after Aries	5 10	06 04 N.
Ditto right Shoulder	85 04 after Aries	5 40	07 19 N.
Secondly, the right ascension, &c. after either of the solstices.			
Great Dog Sirius	8° 16′ after Cancer	00^h 33′	16° 20′ S.
Little Dog	21 14 after Cancer	01 25	05 54 N.
+ Aquila	24 20 after Capricorn	01 37	08 10 N.
Lion's Heart	58 26 after Cancer	03 54	13 17 N.
+ Fomabant	70 35 after Capricorn	04 42	31 03 S.
+ Ala Pegasi	72 46 after Capricorn	04 51	13 44 N.

For the other stars, not comprehended in this Quadrant, recourse may be had for their right ascension, to Mr. Flamstead's Historia Cœlestis, or to 2d Hodgson, 516, or else they must be sought for, by such rules as in other books, or hereafter are given, for that purpose.

 And

And now to proceed to find the time of the ſtar's ſouthing, in the following inſtances.

PROBLEM I.

Prob. 1. *At what time does Arcturus come to his ſouthing the firſt day of April, 1726, the year referred to by Mr. Hodgſon?*

2d Hodgſon, 458. The right aſcenſion of Arcturus (by the preceding table, and by the tables 2d Hodgſon, 518) is 210° 47′ 42″, which, reduced to time, neglecting the 42′, is 14 hours 3 minutes, and, rejecting 12 hours, is 2 hours 3 minutes, agreeing nearly with the hours cut by the thread in the lower annulus on the Quadrant, for his right aſcenſion from the preceding equinoctial point; and whether we conſider the right aſcenſion of Arcturus as 2 hours or 14, the ſame thing will reſult, as hereafter will be ſhewn.

Now, from the right aſcenſion of Arcturus — —	2ʰ 03′
Subtract the ſun's right aſcenſion this day, which, by laying the thread on the day of the month on the upper arcs of months, will fall on 21 degrees, nearly, on the left ſide of the limb of the Quadrant, and the ſame, converted into time, without any addition to it (becauſe the ſun is in the firſt Quadrant) is — — — —	1 24
Which deducted from the ſtar's right aſcenſion, leaves for the ſtar's ſouthing after midnight — — — —	0 39

Or, if for the accommodating the practice in this caſe, to the method preſcribed by Mr. Hodgſon (Vol. II. 458,) we ſet the right aſcenſion of Arcturus on the ſame 1ſt of April, at —	14ʰ 03′
From hence ſubtract the ſun's right aſcenſion — —	1 24
Remains	12 39
And, rejecting 12, there remains, as above, for the ſouthing of Arcturus — — — — — —	0 39

In the preceding caſe the ſun's right aſcenſion is leſs than the ſtar's; let us now ſee how the caſe ſtands when it exceeds it.

PROBLEM II.

At what time does Arcturus transit the meridian the 25th of January, 1727, in the latitude 51° 32′. Prob. 2.

Here, because the sun's right ascension is greater than the star's, therefore, to the right ascension of Arcturus, given by Mr. Hodgson, and agreeing herein nearly with the Quadrant, as before (2d *Hodgson*, 458, 518.)

	02^h	$03'$
Add, to make a subtraction, — — — — — —	24	00
And the sum is	26	03
From this sum take 40°, the sun's right ascension, as found by the Quadrant, with the addition of 270°, because the sun is in the 4th quarter, making together 319°, equal in time to — — — — — — —	21	16
And there remains for the star's southing	4	47

PROBLEM III.

To find the ascensional difference of the same star Arcturus, his declination being according to the circle of the star's declination on the Quadrant, 20° 40′ N. Prob. 3.

The proportion will be, As the co-tangent of the latitude (51° 32′ =) 38° 28′, to the radius, so is the tangent of the declination, 20° 40′; or, according to Mr. Hodgson, 20° 39′, to the sine of the ascensional difference, 28° 18′. (2d *Hodgson*, 461.)

The Practice on the QUADRANT.

Take 38° 28′ from the tangents, in the compasses; with that extent, enter one foot at radius on the sines, and bring the thread to the other; then take 20° 39′ from the tangents, in the compasses, and, with that extent, entering between the thread and the scale of sines, they will rest at 28° 18′, the star's ascensional difference; which, in time, is 1 hour 53 minutes.

The

The above aſcenſional difference, 28° 18′, added to 90° (equal in time to 6 hours) gives 118° 18′ (or 7 hours 53 minutes in time) for the arc of the equator, contained between the ſtar's riſing and his coming to the meridian, being half the time of the ſtar's continuance above the horizon; whence the ſaid time, 7 hours 53′, added to the ſtar's ſouthing, will give the time of his ſetting, and, ſubtracted, will leave the time of its riſing.

2d *Hodgſon*, 461.

12	39
7	53
4	46

Star's riſing.

Now, on the firſt of April, 1726, Arcturus paſſes over the meridian at 12 hours 39′, after mid-day; take from this the ſemi-viſible arc of duration, 7 hours 53′, and it leaves the time of the ſtar's riſing, 4 hours 46′ *propè*.

12	39
7	53
20	32

Setting.

Again, to the time of the tranſit of Arcturus over the meridian, 12 hours 39′, add the ſame ſemi-viſible arc, 7 hours 53′, and it gives the time of his ſetting, 20 hours 32′; which, according to the vulgar way of reckoning, rejecting 12 hours, is 8 hours 32′ in the morning.

And, becauſe the declination of the ſame ſtar is very nearly the ſame during the whole year, it follows, that in the ſame place, the aſcenſional difference will be nearly the ſame, and, conſequently, the ſtar's ſemi-viſible arc, or half of its continuance above the horizon in the ſame place, will be very nearly the ſame; and, therefore, to find the time of the ſtar's riſing or ſetting at any other day of the ſame year, we have nothing more to do, than to find the time of the ſtar's coming to the meridian.

Example I.

2d *Hodgſon*, 462.

Suppoſe it was required to find at what time the ſame ſtar Arcturus, will riſe and ſet at London the 25th of January, 1727, the time referred to by Mr. Hodgſon.

By the operation in page 109, it appeared, that Arcturus would ſouth on the given day at 4 hours 47 minutes; to this add, in order to make a ſubtraction, $12^h\ 0'$, and it will become $16^h\ 47'$; then, from the tranſit of Arcturus, $16^h\ 47'$, take its ſemi-continuance, before found, $7^h\ 53'$, and there remains for his *riſing* that day, $8^h\ 54'$; to the time of his tranſit, $16^h\ 47'$, add the above $7^h\ 53'$, for his *ſetting*, and it gives $24^h\ 40'$, which, according to the aſtronomical way of computing time, is January 26, at $0^h\ 40'$, but, according to the common way, is January 26, at $0^h\ 40'$ in the afternoon.

Ex-

EXAMPLE II.

Let it be required to find the time of the tranſit of Cor. Leonis, over the meridian, on the 25th of December; his declination being north 13° 25′; his right aſcenſion in time 9^h 50′; and the ſun's right aſcenſion, 286°, equal, in time, to 19^h.4′.

Since the right aſcenſion of the ſtar is leſs than that of the ſun, therefore, to the right aſcenſion of the ſtar, 9^h 50′, add 24^h 00′, the ſum is 33^h 50′; from whence ſubtract the ſun's right aſcenſion, 19^h 4′, and the remainder 14^h 46′, is the time of the ſtar's ſouthing, or paſſing over the meridian. Then, to find the aſcenſional difference, ſay, As the co-tangent of the latitude, 38° 28′, to the radius, ſo is the tangent of the declination, 13° 25′, to the ſine of the aſcenſional difference, (17° 43′=) 1^h 11′, *propè*, which may be worked in the common way by the Quadrant: To the aſcenſional difference, equal, in time, to 1^h 11′, add 90°, or 6^h, and the ſum 7^h 11′, is half the time of the ſtar's continuance above the horizon. Tranſit. Aſcenſional difference.

From the above time of the ſtar's tranſit over the meridian, *viz.* 14^h 46′, ſubtract the ſemi-viſible arc of duration, 7^h 11′, and it leaves the time of the ſtar's riſing, 7^h 35′; and, to the time of the tranſit, 14^h 46′, add the ſame ſemi-viſible arc, 7^h 35′, and it gives the time of his ſetting, 22^h 21′. Star's riſing. Setting.

PROBLEM IV.

Given the latitude of the place, ſuppoſe 51° 32′; the time, the firſt of April, 1726; Arcturus, the ſtar; and his ſouthing, 14 hours 3 minutes, at that time; and his diſtance from the north pole, or the complement of his declination, 20° 40′; thence to find the ſtar's amplitude. Prob. 4. 2d *Hodgſon*, 459, 460.

The proportion will be the ſame, as in finding the amplitude of the ſun, *viz.* As the co-ſine of the latitude, (51° 32′=) 38° 28′, to the radius, ſo is the ſine of the ſtar's declination, 20° 40′, to the ſine of the amplitude, 34° 34′. Amplitude.

To be practiſed on the Quadrant in the uſual manner.

The amplitude of the ſtar being always according to the declination, which, in this caſe, is northerly; therefore, it riſes 34° 34′, to the northward of the eaſt point of the horizon, and ſets 34° 34′, to the northward of the weſt, *i. e.* it riſes north eaſt by north, and ſets north weſt by north, nearly.

2d *Hodgſon*, 460.

And, ſince the declination of the ſtars, with reſpect to common uſe, may be ſaid to be conſtantly the ſame, inaſmuch as the ſtars that are near the equinoctial colure, don't alter their declination ſcarce one third of a minute in a year, and thoſe that are near the ſolſtitial colure don't alter their declination the ſame quantity, in an hundred years, and the other intermediate ſtars in proportion, it follows, that the ſame ſtar in the ſame place, has conſtantly the ſame amplitude, and, during the whole year, riſes and ſets in the ſame points of the horizon, nearly.

And, therefore, for thoſe ſtars whoſe declination is given on the Quadrant, we have nothing more to do to find the amplitude, than to ſay (according to the foregoing rule) As the co-ſine of the latitude, to the radius, ſo is the ſine of the declination of the ſtar, to the ſtar's amplitude; to be work'd in the common way.

PROBLEM V.

Prob. 5. *The latitude and longitude of a ſtar, not mention'd on the Quadrant, being given; to find its right aſcenſion and declination.*

EXAMPLE.

The longitude of Pollux being, by the tables, 19° 26′, from Cancer, or 109° 26′ from Aries; its latitude 6° 40′; and the diſtance between the poles of the equator and ecliptic, 23° 29′; thence to find the ſtar's right aſcenſion and declination.

2d *Hodgſon*, 419, 420. The proportion is (according to the eleventh caſe of oblique angled ſpherical triangles,) As radius, to the ſine of the longitude from Aries, 109° 26′ (its ſupplement 70° 34′,) ſo is the co-tangent

of

of the latitude, 6° 40′, to the tangent of a fourth arc, 82° 56′. See *Hawney*, 371.
Now, to bring this within the compass of the Quadrant, let the second term take place of the first, and this will infer a change of the tangents in the third and fourth terms, into their co-tangents; and the proportion will stand thus, As the sine of the longitude, 70° 34′, to the radius, so is the tangent of the latitude, 6° 40′, to the co-tangent of a fourth arc, 7° 4′, whose complement is 82° 56′= the fourth arc, out of which subtract the distance between the two poles, 23° 29′, and the remainder is 59° 27′, a fifth arc.

Then say, As the sine of the fifth arc, 59° 27′, is to the sine of the fourth arc, 82° 56′, so is the co-tangent of the longitude (70° 34″=) 19° 26′, to the tangent of the right ascension from Cancer, 22° 7′; to which add (as the star is in the second Quadrant) 90°, and you have the right ascension from Aries, 112° 7′.

The Practice on the QUADRANT, *in the first proportion.*

Take 70° 34′ from the sines, in the compasses; with that extent, enter one foot at radius on the tangents, and bring the thread to the other; then take 6° 40′, from the tangents, in the compasses, and, with that extent, entering between the thread and the scale of tangents, they will rest at 7° 4′, the complement of 82° 56′, the fourth arc.

Then, in the second proportion.

Take 59° 27′ from the sines, in the compasses; with that extent, enter one foot at the sine of 82° 56′, and bring the thread to the other; then take 19° 26′ from the centre of the line of tangents, in the compasses, and entering that extent between the scale and the thread, they will rest at the tangent of 22° 7′.

To find the declination of the star in the preceding case.

Here the right ascension of the star being found, as before, to be 22° 7′ from Cancer, say, As the sine of the right ascension, 22° 7′, to the sine of 83° 20′, the complement of the latitude, 6° 40′, so is the sine of the longitude from Cancer, 19° 26′, to the sine of 2d *Hodgson*, 420. *Hawney*, 371.

Q 61°,

61° 21′, the complement of 28° 39′, the declination, which is northerly, becauſe the ſtar's place is in the northern half of the ecliptic, and the latitude north.

Note, the longitude and latitude of a fixed ſtar being given, as in the foregoing inſtance, the declination may be found before the right aſcenſion, according to the rules given in 2d Hodgſon, page 420.

PROBLEM VI.

Prob. 6. *Given the right aſcenſion and declination of a ſtar, not marked in the Quadrant; to find his longitude and latitude.*

EXAMPLE.

Let the ſtar be Pollux, the ſouthern ſtar of the twins, whoſe right aſcenſion from Aries (according to 2d Hodgſon, page 516) is 112° 7′. and his declination 28° 39′; thence to find the longitude and latitude.

In this example are given, the conſtant diſtance between the two poles, 23° 29′, the complement of the declination, 61° 21′, and the ſupplement of the ſtar's right aſcenſion, 67° 53′; thence to find the ſtar's longitude and latitude.

2d *Hodgſon*, 423, 197. The proportion will be (by caſe the tenth of oblique ſpherical triangles,) As radius, to the ſine of 22° 7′, the complement of the right aſcenſion of the ſtar, 67° 53′, ſo is the tangent of 61° 21′, the complement of the declination, 28° 39′, to the tangent of a fourth arc. Or, As the co-ſine of (22° 7′=) 67° 53′, is to radius, ſo is the tangent of the declination, 28° 39′, to the tangent of 30° 32′, whoſe complement, 59° 28′, is a fourth arc.

Then, if to the fourth arc — — — —	59°	28′
Be added the conſtant diſtance between the two poles	23	29
The ſum is a fifth arc — — — —	82	57

Again,

Again, ſay, As the co-ſine of a fourth arc, 30° 32′, to the co-ſine of the fifth arc (82° 57′=) 7° 03′, ſo is the ſine of the declination, 28° 39′, to the ſine of the latitude of the ſtar, 6° 40′, which is north becauſe the ſtar's right aſcenſion is leſs than 180°, and its declination north.

Again, to find the longitude, the proportion will be (by the latter part of the ninth caſe of oblique ſpherical triangles,) As the ſine of the fifth arc, 82° 57′, to the ſine of the fourth arc, 59° 28′, ſo is the co-tangent of the right aſcenſion, 22° 7′, to the co-tangent of the longitude, 19° 26′.

2d *Hodgſon*, 423, 194.

All which proportions may be worked in the common way, by the Quadrant.

THE END.

ERRATA.

Page 47, the laſt line but three, for 17° 21′, read 70° 21′. Page 49, the laſt line but two, cancel the letter (P). Page 49, the laſt line but one, for 70° 20′, read 70° 21′, the co-declination. And in page 50, inſtead of 70° 20′, read 70° 21′, and make the needful alterations of one minute in the conſequent numbers. In page 62, line 16, for 7° 47′, read 107° 47′.

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